

Statistical Properties of Share Volume Traded in Financial Markets

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We quantitatively investigate the ideas behind the often-expressed adage ‘it takes volume to move stock prices’, and study the statistical properties of the number of shares traded $Q_{\Delta t}$ for a given stock in a fixed time interval Δt . We analyze transaction data for the largest 1000 stocks for the two-year period 1994-95, using a database that records every transaction for all securities in three major US stock markets. We find that the distribution $P(Q_{\Delta t})$ displays a power-law decay, and that the time correlations in $Q_{\Delta t}$ display long-range persistence. Further, we investigate the relation between $Q_{\Delta t}$ and the number of transactions $N_{\Delta t}$ in a time interval Δt , and find that the long-range correlations in $Q_{\Delta t}$ are largely due to those of $N_{\Delta t}$. Our results are consistent with the interpretation that the large equal-time correlation previously found between $Q_{\Delta t}$ and the absolute value of price change $|G_{\Delta t}|$ (related to volatility) are largely due to $N_{\Delta t}$.

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The distinctive statistical properties of financial time series are increasingly attracting the interest of physicists [1]. In particular, several empirical studies have determined the scale-invariant behavior of both the distribution of price changes [2] and the long-range correlations in the absolute values of price changes [3]. It is a common saying that ‘it takes volume to move stock prices’. This adage is exemplified by the market crash of 19 October 1987, when the Dow Jones Industrial Average dropped 22.6% accompanied by an estimated 6×10^8 shares that changed hands on the New York Stock Exchange alone. Indeed, an important quantity that characterizes the dynamics of price movements is the number of shares $Q_{\Delta t}$ traded (share volume) in a time interval Δt . Accordingly, in this paper we quantify the statistical properties of $Q_{\Delta t}$ and the relation between $Q_{\Delta t}$ and the number of trades $N_{\Delta t}$ in Δt . To this end, we select 1000 largest stocks from a database [4] recording all transactions for all US-stocks, and analyze transaction data for each stock for the 2-year period 1994–95.

First, we consider the time series of $Q_{\Delta t}$ for one stock, which shows large fluctuations that are strikingly non-Gaussian [Fig. 1a]. Figure 1b shows, for each of four actively-traded stocks, the probability distributions $P(Q_{\Delta t})$ which are consistent with a power law decay,

$$P(Q_{\Delta t}) \sim \frac{1}{(Q_{\Delta t})^{1+\lambda}}. \quad (1)$$

When we extend this analysis to each of the 1000 stocks [Fig. 1c,d], we obtain an average value for the exponent $\lambda = 1.7 \pm 0.1$, within the Lévy stable domain $0 < \lambda < 2$.

We next analyze correlations in $Q_{\Delta t}$. We consider the family of correlation functions $\langle [Q_{\Delta t}(t)]^a [Q_{\Delta t}(t + \tau)]^a \rangle$, where the parameter a ($< \lambda/2$) is required to ensure that

the correlation function is well defined. Instead of analyzing the correlation function directly, we apply detrended fluctuation analysis [5], which has been successfully used to study long-range correlations in a wide range of complex systems [6]. We plot the detrended fluctuation function $F(\tau)$ as a function of the time scale τ . Absence of long-range correlations would imply $F(\tau) \sim \tau^{0.5}$, whereas $F(\tau) \sim \tau^\delta$ with $0.5 < \delta \leq 1$ implies power-law decay of the correlation function,

$$\langle [Q_{\Delta t}(t)]^a [Q_{\Delta t}(t + \tau)]^a \rangle \sim \tau^{-\kappa}; \quad [\kappa = 2 - 2\delta]. \quad (2)$$

For the parameter $a = 0.5$, we obtain the average value $\delta = 0.83 \pm 0.02$ for the 1000 stocks [Fig. 2a,b]; so from Eq. (2), $\kappa = 0.34 \pm 0.04$ [7].

To investigate the reasons for the observed power-law tails of $P(Q_{\Delta t})$ and the long-range correlations in $Q_{\Delta t}$, we first note that

$$Q_{\Delta t} \equiv \sum_{i=1}^{N_{\Delta t}} q_i, \quad (3)$$

is the sum of the number of shares q_i traded for all $i = 1, \dots, N_{\Delta t}$ transactions in Δt . Hence, we next analyze the statistical properties of q_i . Figure 3a shows that the distribution $P(q)$ for the same four stocks displays a power-law decay $P(q) \sim 1/q^{1+\zeta}$. When we extend this analysis to each of the 1000 stocks, we obtain the average value $\zeta = 1.53 \pm 0.07$ [Fig. 3b].

Note that ζ is within the stable Lévy domain $0 < \zeta < 2$, suggesting that $P(q)$ is a positive (or one-sided) Lévy stable distribution [8,9]. Therefore, the reason why the distribution $P(Q_{\Delta t})$ has similar asymptotic behavior to $P(q)$, is that $P(q)$ is Lévy stable, and $Q_{\Delta t}$ is related to q through Eq. (3). Indeed, our estimate

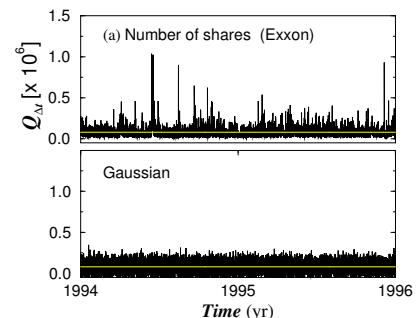
of ζ is comparable within error bounds to our estimate of λ . We also investigate if the q_i are correlated in “transaction time”, defined by i , and we find only “weak” correlations (the analog of δ has a value = 0.57 ± 0.04 , close to 0.5).

To confirm that $P(q)$ is Lévy stable, we also examine the behavior of $Q_n \equiv \sum_{i=1}^n q_i$. We first analyze the asymptotic behavior of $P(Q_n)$ for increasing n . For a Lévy stable distribution, $n^{1/\zeta} P([Q_n - \langle Q_n \rangle]/n^{1/\zeta})$ should have the same functional form as $P(q)$, where $\langle Q_n \rangle = n \langle q \rangle$ and $\langle \dots \rangle$ denotes average values. Figure 4a shows that the distribution $P(Q_n)$ retains its asymptotic behavior for a range of n — consistent with a Lévy stable distribution. We obtain an independent estimate of the exponent ζ by analyzing the scaling behavior of the moments $\mu_r(n) \equiv \langle |Q_n - \langle Q_n \rangle|^r \rangle$, where $r < \lambda$ [10]. For a Lévy stable distribution $[\mu_r(n)]^{1/r} \sim n^{1/\zeta}$. Hence, we plot $[\mu_r(n)]^{1/r}$ as a function of n [Fig. 4b,c] and obtain an inverse slope of $\zeta = 1.45 \pm 0.03$ — consistent with our previous estimate of ζ [11].

Since the q_i have only weak correlations (the analog of δ has the value = 0.57), we ask how $Q_{\Delta t} \equiv \sum_{i=1}^{N_{\Delta t}} q_i$ can show much stronger correlations ($\delta = 0.83$). To address this question, we note that (i) $N_{\Delta t}$ is long-range correlated [14], and (ii) $P(q)$ is consistent with a Lévy stable distribution with exponent ζ , and therefore, $N_{\Delta t}^{1/\zeta} P([Q_{\Delta t} - \langle q \rangle N_{\Delta t}]/N_{\Delta t}^{1/\zeta})$ should, from Eq. (3), have the same distribution as any of the q_i . Thus, we hypothesize that the dependence of $Q_{\Delta t}$ on $N_{\Delta t}$ can be separated by defining $\chi \equiv [Q_{\Delta t} - \langle q \rangle N_{\Delta t}]/N_{\Delta t}^{1/\zeta}$, where χ is a one-sided Lévy-distributed variable with zero mean and exponent ζ [8,9]. To test this hypothesis, we first analyze $P(\chi)$ and find similar asymptotic behavior to $P(Q_{\Delta t})$ [Fig. 4d]. Next, we analyze correlations in χ and find only weak correlations [Fig. 4e,f] — implying that the correlations in $Q_{\Delta t}$ are largely due to those of $N_{\Delta t}$.

An interesting implication is an explanation for the previously-observed [12,13] equal-time correlations between $Q_{\Delta t}$ and volatility $V_{\Delta t}$, which is the local standard deviation of price changes $G_{\Delta t}$. Now $V_{\Delta t} = W_{\Delta t} \sqrt{N_{\Delta t}}$, since $G_{\Delta t}$ depends on $N_{\Delta t}$ through the relation $G_{\Delta t} = W_{\Delta t} \sqrt{N_{\Delta t}} \epsilon$, where ϵ is a Gaussian-distributed variable with zero mean and unit variance and $W_{\Delta t}^2$ is the variance of price changes due to all $N_{\Delta t}$ transactions in Δt [14]. Consider the equal-time correlation, $\langle Q_{\Delta t} V_{\Delta t} \rangle$, where the means are subtracted from $Q_{\Delta t}$ and $V_{\Delta t}$. Since $Q_{\Delta t}$ depends on $N_{\Delta t}$ through $Q_{\Delta t} = \langle q \rangle N_{\Delta t} + N_{\Delta t}^{1/\zeta} \chi$, and the equal-time correlations $\langle N_{\Delta t} W_{\Delta t} \rangle$, $\langle N_{\Delta t} \chi \rangle$, and $\langle W_{\Delta t} \chi \rangle$ are small (correlation coefficient of the order of ≈ 0.1), it follows that the equal-time correlation $\langle Q_{\Delta t} V_{\Delta t} \rangle \propto \langle N_{\Delta t}^{3/2} \rangle - \langle N_{\Delta t} \rangle \langle N_{\Delta t}^{1/2} \rangle$, which is positive due to the Cauchy-Schwartz inequality. Therefore, $\langle Q_{\Delta t} V_{\Delta t} \rangle$ is large because of $N_{\Delta t}$.

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- [7] Here, κ is the exponent characterizing the decay of the autocorrelation function, compactly denoted $\langle [Q_{\Delta t}(t)]^a [Q_{\Delta t}(t+\tau)]^a \rangle$. Values of a in the range $0.1 < a < 1$ yield δ in the range $0.75 < \delta < 0.88$ — consistent with long-range correlations in $Q_{\Delta t}$.
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- [10] The values of ζ reported are using $r = 0.5$. Varying r in the range $0.2 < r < 1$ yields similar values.
- [11] To avoid the effect of weak correlations in q on the estimate of ζ , the moments $[\mu_r(n)]^{1/r}$ are constructed after randomizing each time series of q_i . Without randomizing, the same procedure gives an estimate of $\zeta = 1.31 \pm 0.03$.
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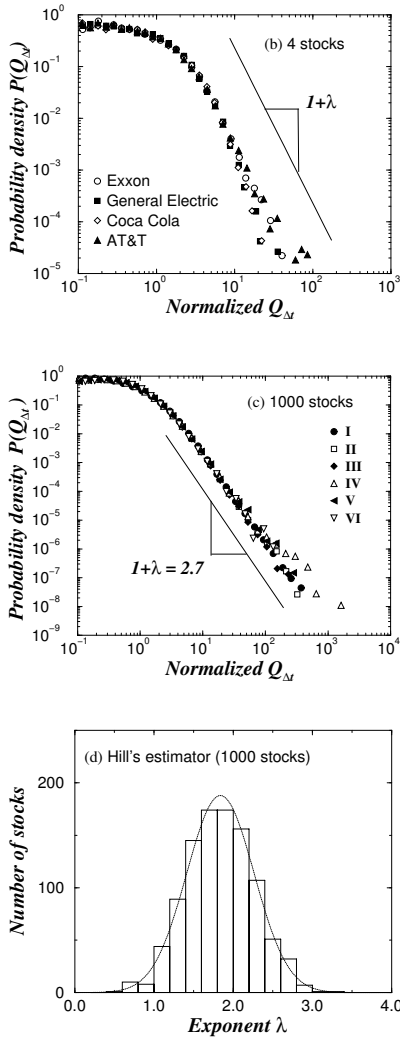


FIG. 1. (a) Number of shares traded [15] for Exxon Corporation (upper panel) for an interval $\Delta t = 15$ min compared to a series of Gaussian random numbers with the same mean and variance (lower panel). (b) Probability density function $P(Q_{\Delta t})$ for 4 actively-traded stocks Exxon Corp., General Electric Co., Coca Cola Corp., and A T & T Corp., shows an asymptotic power-law behavior characterized by an exponent $1 + \lambda$. Hill's method [16] gives $\lambda = 1.87 \pm 0.14, 2.10 \pm 0.17, 1.91 \pm 0.20,$ and 1.71 ± 0.09 respectively. (c) $P(Q_{\Delta t})$ for 1000 stocks on a log-log scale. To choose compatible sampling time intervals Δt , we first partition the 1000 companies studied into six groups [14] denoted I - VI, based upon the average time interval between trades δt . For each group, we choose $\Delta t > 10 \delta t$, to ensure that each interval has a sufficient $N_{\Delta t}$. Thus we choose $\Delta t = 15, 39, 65, 78, 130$ and 390 min for groups I - VI respectively, each containing ≈ 150 companies. Since the average value of $Q_{\Delta t}$ differs from one company to the other, we normalize $Q_{\Delta t}$ by its average. Each symbol shows the probability density function of normalized $Q_{\Delta t}$ for all companies that belong to each group. Power-law regressions on the density functions of each group yield the mean value $\lambda = 1.78 \pm 0.07$. (d) Histogram of exponents λ_i for $i = 1, \dots, 1000$ stocks obtained using Hill's estimator [16], shows an approximately Gaussian spread around the average value $\lambda = 1.7 \pm 0.1$.

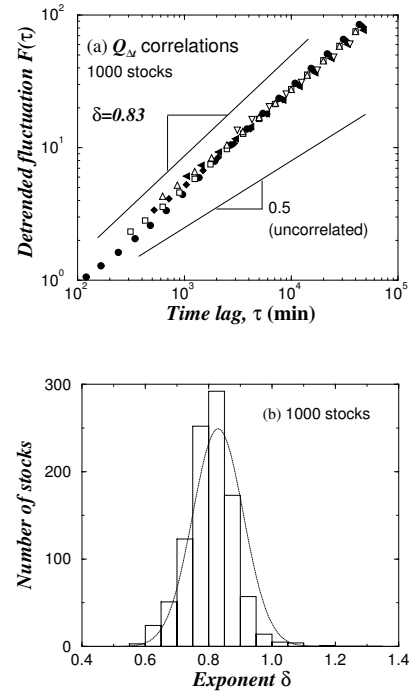
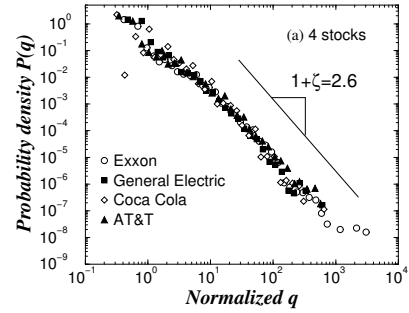


FIG. 2. (a) Detrended fluctuation function $F(\tau)$ for $(Q_{\Delta t})^a$ for $a = 0.5$ [7], averaged for all stocks within each group (I-VI) as a function of the time lag τ . $F(\tau)$ for a time series is defined as the χ^2 deviation of a linear fit to the integrated time series in a box of size τ [5]. An uncorrelated time series displays to $F(\tau) \sim \tau^\delta$, where $\delta = 0.5$, whereas long-range correlated time series display values of exponent in the range $0.5 < \delta \leq 1$. In order to detect genuine long-range correlations, the U-shaped intraday pattern for $Q_{\Delta t}$ is removed by dividing each $Q_{\Delta t}$ by the intraday pattern [3]. (b) Histogram of δ obtained by fitting $F(\tau)$ with a power-law for each of the 1000 companies. We obtain a mean value of the exponent 0.83 ± 0.02 .



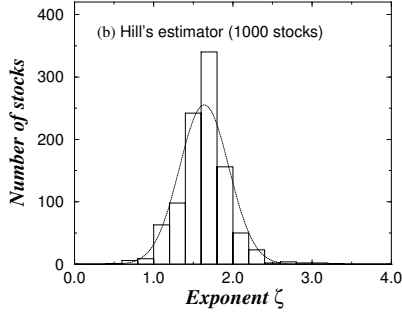


FIG. 3. (a) Probability density function of the number of shares q_i traded, normalized by the average value, for all transactions for the same four actively-traded stocks. We find an asymptotic power-law behavior characterized by an exponent ζ . Fits yield values $\zeta = 1.87 \pm 0.13, 1.61 \pm 0.08, 1.66 \pm 0.05, 1.47 \pm 0.04$, respectively for each of the 4 stocks. (b) Histogram of the values of ζ obtained for each of the 1000 stocks using Hill's estimator [16], whereby we find the average value $\zeta = 1.53 \pm 0.07$.

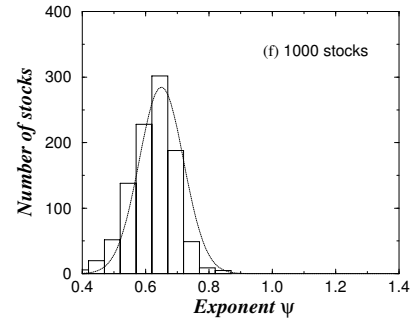
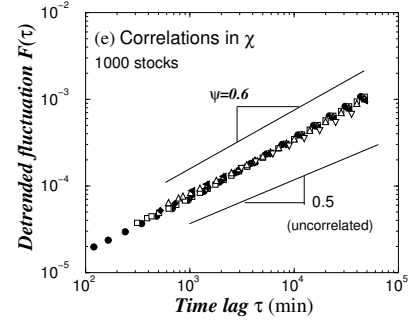
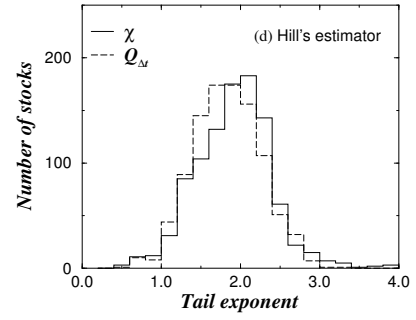
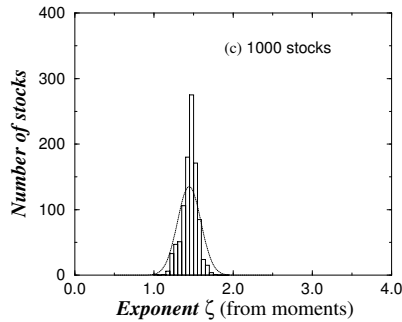
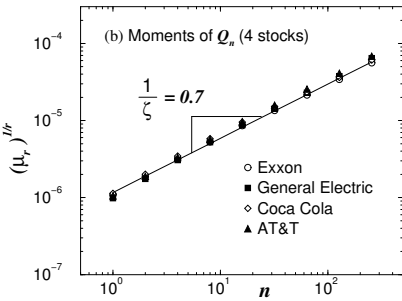
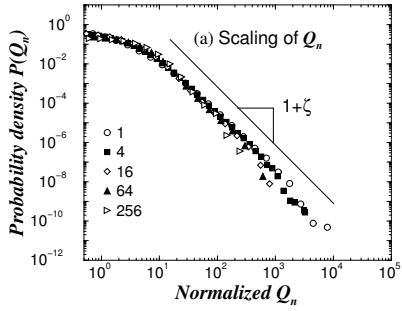


FIG. 4. (a) Probability distribution of Q_n as a function of increasing $n = 1, \dots, 256$ apparently retains the same asymptotic behavior. (b) Scaling of the r^{th} moments μ_r with increasing n for the same four stocks. The inverse slope of this line yields an independent estimate of the exponent ζ . We obtain $\zeta = 1.43 \pm 0.02, 1.35 \pm 0.03, 1.42 \pm 0.01, 1.41 \pm 0.02$ respectively. (c) Histogram of exponents ζ obtained by fitting a power-law to the equivalent of part (b) for all 1000 stocks studied. We thus obtain a value $\zeta = 1.45 \pm 0.03$ consistent with our previous estimate using Hill's estimator. (d) Histogram of slopes estimated using Hill's estimator for the scaled variable $\chi \equiv [Q_{\Delta t} - \langle q \rangle N_{\Delta t}] / N_{\Delta t}^{1/\zeta}$ compared to that of $Q_{\Delta t}$. We obtain a mean value 1.7 ± 0.1 for the tail exponent of χ , consistent with our estimate of the tail exponent λ for $Q_{\Delta t}$. (e) Detrended fluctuation function $F(\tau)$ for χ , where each symbol denotes an average of $F(\tau)$ for all stocks within each group (I-VI as in Fig. 1). (f) Histogram of detrended fluctuation exponents for χ . We obtain an average value for the exponent 0.61 ± 0.03 which indicates only weak correlations compared to the value of the exponent $\delta = 0.83 \pm 0.03$ for $Q_{\Delta t}$.