Differences of Opinion Make a Horse Race

Milton Harris, Artur Raviv

Review of Financial Studies, Volume 6, Issue 3 (1993), 473-506.

Differences of Opinion Make a Horse Race

Milton Harris
University of Chicago

Artur Raviv
Northwestern University

A model of trading in speculative markets is developed based on differences of opinion among traders. Our purpose is to explain some of the empirical regularities that have been documented concerning the relationship between volume and price and the time-series properties of price and volume. We assume that traders share common prior beliefs and receive common information but differ in the way in which they interpret this information. Some results are that absolute price changes and volume are positively correlated, consecutive price changes exhibit negative serial correlation, and volume is positively autocorrelated.

We develop a model of trading in speculative markets based on announcements of public information. Our purpose is to explain some of the empirical regular-

"It were not best that we should all think alike; it is difference of opinion that makes horse races" (Mark Twain in *Pudd'nhead Wilson,* 1894). Milton Harris is the Chicago Board of Trade Professor of Finance and Business Economics at the University of Chicago. Artur Raviv is the Alan E. Peterson Professor of Finance, Northwestern University. The authors gratefully acknowledge the financial support of the Bradley Foundation. Professor Harris wishes to acknowledge partial financial support from Dimensional Fund Advisors. The authors also thank Yakov Amihud, Stephen Blough, Peter DeMarzo, Michael Fishman, Larry Harris, Ronen Israel, Charles Kahn, Robert Korajczyk, Josef Lakonishok, Mark Mitchell, Katherine Schipper, Bill Schwert, Chester Spatt (the executive editor), Lars Stole; and Paul Pleiderer, the referee, for valuable comments, as well as seminar participants at the Federal Reserve Board, The Economics of Financial Markets Conference at Johns Hopkins University (April 1991), American Finance Association Meetings (January 1992), Michigan, Berkeley, Stanford, MIT, Dartmouth, USC, and Rochester. Address correspondence to Milton Harris, Graduate School of Business, University of Chicago, 1101 East 58th Street, Chicago IL 60637.

The Review of Financial Studies 1993 Volume 6, number 3, pp. 473–506

ities that have been documented concerning the relationship between volume and price and the time-series properties of these two variables. We view this goal as an important step in understanding the operation of financial markets.

It is apparent that financial markets respond to public announcements of relevant information. This fact is amply documented by countless event studies that measure the price reaction to announcements of earnings, dividends, new issues, macroeconomic data, and so on. Trading volume has been used less frequently to measure the market response to announcements; however, several papers have shown that trading volume responds to earnings announcements. Beaver (1968, p. 91), for example, shows that volume in the week of an earnings announcement is, on average, about 34 percent higher than average volume over the corresponding nonreport period.[1]

Explanations for trading volume include tax-driven trading, liquidity trading, portfolio rebalancing, and speculation. We focus on speculative trading as being the major factor accounting for surges in market activity following public announcements. Speculative trading stems, presumably, from disagreements among traders over the relationship between the announcement and the ultimate performance of the assets in question. Such disagreements can arise either because speculators have different private information or because they simply interpret commonly known data differently.

Since the private information approach has been explored more extensively in the literature (but without accounting for many of the stylized facts, see below), we adopt the second approach. Thus, we assume that traders receive common information but differ in the way in which they interpret this information, and each trader believes absolutely in the validity of his or her interpretation. We refer to this as the assumption that traders have "differences of opinion." It seems to us that people often share common information yet disagree as to the meaning of this information, not only in the evaluation of risky assets but also in the evaluation of political candidates, economic policies, and the outcomes of horse races. One example is the variety of opinions among financial analysts and macroeconomists regarding future movements of interest rates, exchange rates, gross national product, and stock prices despite the fact that all these analysts have access to the same economic data. Another example is the fact that weather forecasts often differ even though all forecasters have access to the same National Weather Service data.

We assume that traders start with common prior beliefs about the returns to a particular asset. As information about the asset becomes

[1] See also Bamber (1987) and Ziebart (1990).

available, each trader updates his beliefs about returns using his *own* model (likelihood function) of the relationship between the news and the asset's returns. We assume that there are two types of risk-neutral, speculative traders. The two types agree on whether a given piece of information is favorable or unfavorable, but they disagree on the *extent* to which the information is important. Speculators in the "responsive" group increase (decrease) their probability of high returns more upon receipt of favorable (unfavorable) information than those in the "unresponsive" group. Therefore, when the cumulative impact of the past information is favorable, the responsive speculators value the asset more highly and will own all of it. On the other hand, when the cumulative impact of the past information is unfavorable, the unresponsive speculators value the asset more highly and will own all of it. Thus, trading occurs when, and only when, cumulative information switches from favorable to unfavorable, or vice versa.

Our main results are as follows:

- Absolute price changes and volume are positively correlated.
- Absolute changes in the mean forecast of the final payoff and volume are positively related.
- If speculators overestimate (underestimate) the true quality of the signal, then consecutive price changes exhibit negative (positive) serial correlation.
- Volume is positively autocorrelated.
- Volume is larger than usual on average at the opening of the market after any period in which it is closed (e.g., overnight, weekends, and holidays).

The literature on asset markets has, to a great extent, focused on explaining asset prices with only peripheral attention to trading volume. Two branches of the literature concern us: rational expectations asset pricing models and models based on differences of opinion.[2] Rational expectations models generate disagreement through private information. These models generally involve trading among privately informed traders, uninformed traders, and liquidity (or noise) traders [see, e.g., Grossman and Stiglitz (1976, 1980), Hellwig (1980), Diamond and Verrecchia (1981), Pfleiderer (1984), Kyle (1985), Admati and Pfleiderer (1988), Grundy and McNichols (1989), Foster and Viswanathan (1990, 1993a), Holthausen and Verrecchia (1990), Kim and Verrecchia (1991a, 1991b), Blume et al. (1991), Wang (1992),

[2] Another line of research focuses on volume caused by tax-driven trading strategies. This research shows, for example, that volume will be higher after a decline in the price of the asset. See, for example, Lakonishok and Smidt (1986).

and Shalen (1993)].[3] There are two major differences between the approach in this literature and our approach. First, in rational expectations models, trade is not generated by *public* information signals.[4] Second, in rational expectations models, disagreement is the result of private information. Consequently, speculators try to infer the information of others from their behavior and from market prices; that is, agents are influenced by the beliefs of others.[5] As a result, the ability of traders to learn from prices and the behavior of other traders must be obscured by noise.[6] Our differences-of-opinion approach allows us to ignore learning from market prices and to dispense with noise traders.

Volume results derived from rational expectations models are difficult to compare with those from our model, because volume in these models includes portfolio rebalancing, liquidity, and speculative trades. Our results concern only speculative trades. Moreover, much of the volume of trade in rational expectations models is driven by *exogenous* shocks, for example, to liquidity traders' demands or to endowments, or by exogenous assumptions about the volume of trade *among* liquidity traders. Nevertheless, some comparisons can be made. Similar to our result, Pfleiderer (1984) shows that successive price changes are negatively correlated, although he attributes this result in his model to the assumed nature of liquidity trading (see p. 17). Pfleiderer (1984), Blume et al. (1991), and Wang (1992) show a positive contemporaneous covariance between volume and absolute price changes. Shalen (1993) shows that dispersion of beliefs contributes to this covariance. In Pfleiderer's model, however, this result is due entirely to nonspeculative trading, because the correlation between speculative trading volume and absolute price changes is zero (see p. 18). Foster and Viswanathan (1993a) and Wang (1992) also show that trading volume has positive serial correlation. Admati

[3] Diamond and Verrecchia (1981) do not include liquidity traders. Instead, they postulate random shocks to the endowments of privately informed traders. In this model, volume consists of portfolio rebalancing and speculative trades. Also, Blume et al. (1991) do not include liquidity traders. In their model, traders have private information about the risky asset and about the precision of this information.

[4] Kim and Verrecchia (1991a, 1991b) and Grundy and McNichols (1989) are exceptions. In these models trade is generated by public information, because traders disagree on its interpretation due to prior private information. Kim and Verrecchia (1991a, 1991b) show that trading volume is proportional both to absolute price change and the extent to which the precision of prior, private information differs across traders. Grundy and McNichols (1989) obtain an expression relating the volume of trade to the strength of the correlation between the errors in the public and private signals and the precisions of the signals.

[5] Thus, noisy rational expectations models with private signals cannot be interpreted as models with common signals and differences of opinion, contrary to the claim in Pfleiderer (1984) and Holthausen and Verrecchia (1990).

[6] Milgrom and Stokey (1982) show that speculative trades based purely on differences in private information cannot occur among risk-averse traders in the absence of noise traders. See Varian (1989) for a short review of this and similar "no-trade" results.

and Pfleiderer (1988) and Foster and Viswanathan (1990) show that volume will tend to be high in some periods and low in adjacent periods. If we interpret periods in our model as being the same as in these two models, then this implication is at variance with our result that volume is positively autocorrelated. On the other hand, if our periods are shorter or longer than their periods, there is no contradiction. Also, Foster and Viswanathan (1990) show that volume can be expected to be lower on Mondays, just the opposite of our result.[7]

The second branch of the literature in which trading is induced by differences of opinion is exemplified by the work of Harrison and Kreps (1978), Varian (1985, 1989), Blough (1988), Hindy (1989), De Long et al. (1990), and Kandel and Pearson (1992). Harrison and Kreps (1978) analyze a model very similar to ours to show that prices can contain a speculative component in addition to reflecting fundamental value. Varian (1985, 1989) focuses on differences in prior beliefs as opposed to differences in models. He shows "the relationship between the *equilibrium* price and volume of trade and the *equilibrium* probability beliefs about those assets" (1989, p. 5). Blough (1988) examines the effect on the informativeness of asset prices when traders have both private information and differences of opinion. The model of Hindy (1989) includes traders who have different models and who also receive both public and private information. He shows, using examples, that this model is capable of producing expected volumes and price changes that are "positively related, negatively related for all time periods, or have a relation that changes from positive to negative, or vice versa, over time" (p. 8). De Long et al. (1990) include two types of traders, one who correctly anticipates future price movements and one whose estimate of future prices is distorted by an additive, random noise term.[8] They show that their model can account for such apparent anomalies as closed-end fund discounts and premia and the equity premium and excess volatility puzzles. The approach in Kandel and Pearson (1992) is very similar to ours, although the specific models are somewhat different. In both models, traders receive only public information. The major differences are that traders in Kandel and Pearson (1992) have *both* different prior beliefs and different likelihood functions and are risk averse. We consider only different models. Moreover, Kandel and Pearson analyze only a two-period model and make different distributional assumptions. Their main results are that volume and absolute

[7] Their result follows from the assumptions that private information accrues over weekends, but public information does not, and that liquidity traders can postpone their trades by at most one day.

[8] See also Shleifer and Summers (1990) for a less technical description.

price changes are positively related and that volume can be positive even when price does not change. This last result is in contrast to our model in which volume occurs only when price changes.[9]

To summarize, the theoretical literature has only recently begun to produce results concerning volume. Several articles obtain a result similar to ours on the relation between volume and absolute price changes. Pfleiderer (1984) also obtains our result that consecutive price changes are negatively correlated. Admati and Pfleiderer (1988) and Foster and Viswanathan (1990) show that trading will tend to bunch. Foster and Viswanathan (1993a) and Wang (1992) show that volume is positively autocorrelated. Finally, Foster and Viswanathan (1990) predict lower volume on Mondays. Our results that do not appear in the previous literature are that absolute changes in the mean forecast of the final payoff and volume are positively related, and volume is larger than usual on average at the opening of the market after any period in which it is closed. Moreover, our model produces all these results from a single framework.

Empirical results on volume are fairly abundant. To a large extent, these are consistent with our implications. A number of studies have documented empirically the positive correlation between volume and absolute price changes or price variance over time [see, e.g., Karpoff (1987), Jain and Joh (1988), Schwert (1989), Gallant et al. (1992), and Lang et al. (1992)]. Patell and Wolfson (1984) find, consistent with our result and that of Pfleiderer (1984), that successive transaction price changes are negatively autocorrelated. Evidence that volume tends to be higher in the beginning and end of trading days is provided by Jain and Joh (1988). Amihud and Mendelson (1987, 1991) find evidence of higher volume at the opening of the market. Foster and Viswanathan (1993b) also observe a U-shaped intraday volume pattern and find that trading volume is lower on Mondays.[10] Lakonishok and Maberly (1990), however, point out that volume for individuals is larger on Mondays, and volume for institutions is smaller. Our prediction of higher volume following a period of market closure is consistent with the evidence of high volume at market opening and greater individual trading on Mondays (we discuss this point further later). Also consistent with our implication, Ziebart (1990) documents a positive relation between volume and the absolute change in the mean forecast of analysts. Our prediction of positive autocorrelation in volume is supported by evidence in Harris (1987).

[9] An additional recent model involving differences of opinion is presented in Detemple and Murthy (1992).

[10] Both Foster and Viswanathan (1993b) and Brock and Kleidon (1992), using intraday data, find that volume and trading costs are positively correlated. This is inconsistent with the prediction of Admati and Pfleiderer (1988).

Section 1 presents the formal model in detail. Section 2 derives and interprets the main results regarding the evolution of trading volume and prices. Section 3 briefly considers an extension of the model to the case in which investors are risk averse. Conclusions are contained in Section 4. All formal proofs are relegated to the Appendix.

1. Model

Here we describe a model of trade based on differences of opinion, using the approach discussed in the introduction. There are two groups of risk-neutral speculators who trade, at dates $t = 1,2,...,T,$ shares of an asset that makes a single random payment of $\tilde{R}$ at date T. Shares of the risky asset are traded for a riskless asset. For simplicity, we neglect discounting, so the rate of return on the riskless asset is zero.

The final payoff, $\tilde{R}$, can be either high (H) or low (L, with $L < H$) with equal probabilities.[11] At each date $t = 1,...,T,$ new public information in the form of a signal, $\tilde{s}_t$, is revealed, after which speculators update their beliefs and may trade at a price p_t. Examples of the signals are earnings announcements, macroeconomic data, or political events. The signals are independent and identically distributed conditional on the true payoff. The true probability density of a signal s given a final payoff $R \in \{H, L\}$ is denoted by $\sigma_R^0(s)$.

To clarify the interpretation of the signal and simplify the analysis, we make an assumption that guarantees that the true posterior at date t depends on the history of signals only through the cumulative signal. Formally, we assume

$$\sigma_H^0(s) = \sigma_L^0(-s) = \begin{cases} k_0 a_0^s & \text{for } s \geq 0, \\ k_0 b_0^{-s} & \text{for } s < 0, \end{cases} \tag{1}$$

where a_0 and b_0 are parameters strictly between 0 and 1, and k_0 is a constant required to make $\sigma_H^0(s)$ and $\sigma_L^0(s)$ density functions.[12] Using Bayes' rule and Equation (1), after observing a history of signals $s^t = (s_1,...,s_t)$, we find the true posterior probability that $\tilde{R} = H$ is

$$\pi_{Ht}^0(s^t) = \frac{\Pi_{\tau=1}^t \sigma_H^0(s_\tau)}{\Sigma_{R=H,L} \Pi_{\tau=1}^t \sigma_R^0(s_\tau)} = [1 + \theta_0^m]^{-1} \equiv \pi_H^0(m), \tag{2}$$

[11] Later, we assume that traders realize that the true prior probabilities are equal. This assumption makes the analysis much more tractable. We conjecture that all our results go through if traders share the same probabilities even if they are not one-half.

[12] Additivity requires that the likelihood functions be symmetric, that is, that the probability of a particular signal when the final payoff is high is the same as the probability of minus that signal when the final payoff is low.

where $m = s_1 + \ldots + s_t$ is the *cumulative signal* and $\theta_0 = b_0/a_0$. In other words, conditional independence of the signals over time and our assumption that the likelihood functions are exponential imply that signals are *additive* in the sense that a signal of, say, $s = 5$ is equivalent to 5 signals of 1 each (or a signal of 8 and a signal of -3). Since the posterior depends on the signal history only through the cumulative signal m, we substitute m for s^t and drop the t subscript in the posterior. Obviously, the probability that $\tilde{R} = L$, given m, is $\pi_L^0(m) = 1 - \pi_H^0(m)$.

So that we may interpret larger values of the signal as more favorable information, we assume that the posterior probability of a high outcome is increasing in m (i.e., $\theta_0 < 1$). We may interpret θ_0 as an inverse measure of the quality of the signal. For example, if $\theta_0 = 0$, then any positive signal results in a posterior that assigns probability 1 to $\tilde{R} = H$, and any negative signal results in a posterior that assigns probability 1 to $\tilde{R} = L$. Conversely, for $\theta_0 = 1$, the posterior is independent of the signal.

This completes the description of the true distributions of payoffs and signals. We now consider the beliefs of speculators regarding these elements. All speculators know the correct prior that $\tilde{R}$ can be either H or L with equal probabilities. Differences of opinion are generated by assuming that speculators have different models for interpreting the signals. After observing the signal, each speculator revises his beliefs regarding the final payoff, $\tilde{R}$, using Bayes' rule and his *own* model (likelihood function) of the relation between signals and the final payoff. We assume that each speculator is absolutely convinced that his or her model is correct.[13] Indeed, each group believes the other group is basing its decisions on an incorrect model (i.e., is irrational in this sense).[14] All information, including speculators' models, is assumed to be common knowledge, so speculators make no attempt to infer anything from the behavior of prices or other speculators.

The speculators' models have the same functional form as the true likelihood function but with different parameters. Consequently, the resulting posteriors also exhibit the same functional form as the true posteriors. In particular, speculators realize that the signals are i.i.d.

[13] One could assume that speculators are unsure of the true model and revise their beliefs about it as well as about the final payoff. We neglect this added complication on grounds that the true model is likely to change sufficiently rapidly that such updating is of negligible importance. Thus, the period length and the number of periods, T, in our model should be interpreted as fairly small. Note that absolute belief in a model is not irrational in the Bayesian sense, especially since, in our formulation, there are no possible realizations of the signal that are inconsistent with either group's model (i.e., occur with zero probability according to the model).

[14] Thus, while we maintain the assumption of rational agents, we drop the assumption that rationality is common knowledge.

conditional on the final payoff, but they do not know the true density functions $\sigma_R^0(s)$. Instead, group j ($j = 1, 2$) believes that the conditional density of a signal s given final payoff R is $\sigma_R^j(s)$, where σ_R^j is the same as σ_R^0 with parameters k_j, a_j, and b_j replacing k_0, a_0, and b_0. As before, a_j and b_j are between 0 and 1, and we assume $\theta_j \equiv b_j/a_j < 1$ for $j \in \{1, 2\}$. The posterior probabilities, $\pi_H^j(m)$, derived from the prior and group j's model of the signal-generating process are given by the same formula as the true posteriors [i.e., Equation (2)], with j replacing 0.

Obviously, given the foregoing assumptions, if the groups are to have models resulting in different posterior beliefs, they must have different values of θ_j. As a convention, we assume that $\theta_1 < \theta_2$. Consequently, given the previous interpretation of θ, group 1 believes the signal is of higher quality than group 2 and therefore responds to a given signal history to a greater extent.[15] It follows that group 1 is more optimistic when the cumulative signal is positive and is more pessimistic when m is negative. When $m = 0$, both groups revert to their prior beliefs, namely that the payoff is high with probability $\frac{1}{2}$. Figure 1 depicts $\pi_H^j(m)$ for $j = 1, 2$. We refer to group 1 as the "responsive" group and to group 2 as the "unresponsive" group.

Two important properties of $\pi_H^j(m)$ that follow from (2) are that π_H^j is monotone increasing in m and is concave for $m > 0$ and convex for $m < 0$ (see Figure 1). These properties reflect the facts that larger cumulative signals indicate greater likelihood of high final payoff and that the posterior is more sensitive to changes in cumulative signal when beliefs are more diffuse (i.e., when m is near zero).

We turn now to the speculators' preferences and trading. Recall that all traders are risk neutral. Risk neutrality is appropriate since we are interested primarily in volume generated by speculation as opposed to hedging or life-cycle considerations. Thus, in our model, there is no trading except for speculative purposes. Risk neutrality also implies, however, that demand functions are infinitely elastic. Thus, any trader will seek to buy (sell) an infinite number of shares at any price below (above) his reservation price. Consequently, to make the equilibrium well defined, we must assume that there is a fixed number of shares available. This implies that short sales are not allowed and that, whenever trading occurs, all available shares will be purchased by the more optimistic group. Furthermore, as long as one group remains more optimistic, there will be no further trade. Trade will occur only when the two groups "switch sides." This fact is a direct consequence of our risk-neutrality assumption. In a model

[15] Note that the two groups have access to exactly the same information; that is, neither group has more precise information than the other. One group simply *believes* the signal is more informative than the other group does.

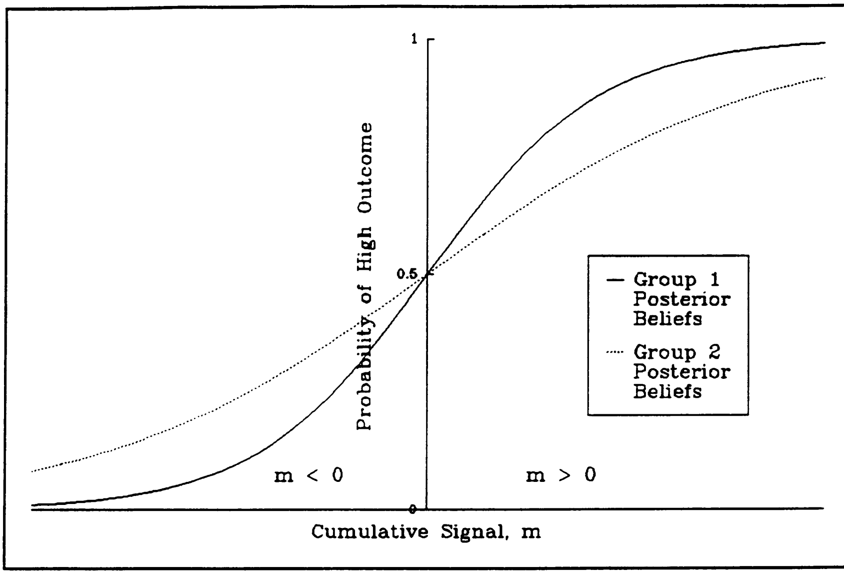

Figure 1
The probability of a high final outcome is shown as a function of the cumulative signal, m, for both groups of speculators. Group 1 is assumed to be more responsive to the signal than group 2. For a positive cumulative signal, group 1 values the asset more highly; the reverse is true for $m < 0$.

with risk-averse speculators, whenever one group's beliefs change relative to the other's some trading will occur. For tractability, our model approximates these trades as negligible.[16]

The price of the risky security changes to reflect the new reservation prices of the traders each time information appears. The reservation price of group j in any period is group j's expectation of next period's equilibrium price.[17] The equilibrium price in any period will be between the two reservation prices for that period, but its precise value will depend on how the market is organized. For tractability, we assume that in every period one group (the same one in each period) has sufficient market power to offer a price on a take-it-or-leave-it basis. This price will equal the "price-taking" group's reservation price.[18] Thus the price-taking group always engages in trades that they believe have zero net present value.[19] It makes no difference

[16] See Section 3 for a discussion of the risk-averse case.

[17] This can be proved by backward induction. See Lemma 2 in the Appendix.

[18] Notice that we are assuming that the market price equals the price taker's reservation price even in periods in which there are no gains to trade. This is innocuous, since, in such periods, any price between the two reservation prices results in no trade.

[19] If one assumes that the equilibrium price is competively set in each period, then the price in any period in which there is trade will equal the reservation price of the buyer. This occurs because the buyer has infinitely elastic demand, but the supply is bounded. Which group is the buyer

which group is the designated price taker as long as it is the same group in each period. Since either group can be the price-taking group, we use symbols without a j subscript or superscript (e.g., $\pi_H(m)$, $\sigma_H(s)$, θ) to denote variables pertaining to the price-taking group.

One interpretation of this assumption is that one group consists mainly of large traders (e.g., institutional investors), and the other group consists mainly of small, individual investors. Under this interpretation, market makers give preferential treatment to the large traders by filling their orders at the reservation price of the small investors.

The price-taking group's reservation price in any period is that group's current expectation of the final payoff.[20] Thus, if the cumulative signal at date t is m, then the reservation price of the price-taking group and the price of the risky asset at date t is

$$p_t = p(m) = H\pi_H(m) + L\pi_L(m) = (H - L)\pi_H(m) + L, \tag{3}$$

where π_H is given by (2). Note that, since the beliefs depend on the signal history only through the cumulative signal, so does the price. Moreover, as is clear from (3), price changes are proportional to changes in the posterior of the price-taking group.

2. Equilibrium Volume and Price Dynamics

Having described a simple model based on heterogeneous beliefs, we now characterize the evolution of prices and volume of trade that results from this model. Recall that the equilibrium price in any period is the reservation price of the price-taking group of speculators in that period, even if at that price no trade takes place. Thus, prices change every period whether or not trading occurs. Our first result shows that these price changes are larger on average in periods of positive volume than in periods of zero volume (see the Appendix for a formal proof).

Theorem 1. *Absolute price changes and speculative volume are positively correlated.*[21]

changes from period to period. Consequently, each group's reservation price in any period will involve that group's expectation of the other group's reservation price in the next period. Therefore, the equilibrium price in any period will involve each group's expectation of the other group's expectation of the first group's expectation, and so forth. This quickly becomes intractable. We are grateful to Charles Kahn for clarifying our thinking on this point.

[20] As mentioned, one's reservation price today is his expectation of tomorrow's equilibrium price. For the price-taking group, tomorrow's equilibrium price is the price taker's expectation in that period of the following period's equilibrium price. The result follows by successive iteration forward to period T and applying the "law of iterated expectations." See Lemma 2 for a formal proof.

[21] Covariances and expectations in all theorems are calculated with the true distribution of the signal. These correspond to the estimates of the covariances and expectations that would be obtained from a time series generated by the model.

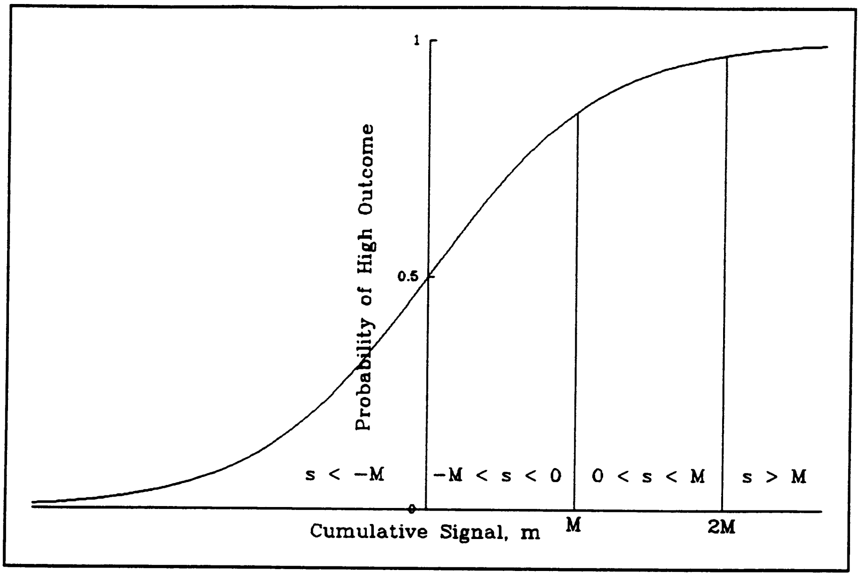

Figure 2
The probability of a high final outcome is shown as a function of the cumulative signal, m. If the current cumulative signal is M, signals $s < -M$ result in positive volume. Signals $s \geq -M$ result in zero volume. Three ranges of such signals are shown. For signals between $-M$ and M, the change in probability of high outcome is smaller in absolute value than the change for $s = -M$. For $s > M$, the change in probability is smaller in absolute value than that corresponding to $-s$.

This result is similar to results in Pfleiderer (1984), Blume et al. (1991), Kim and Verrecchia (1991a, 1991b), Wang (1992), and Kandel and Pearson (1992). A number of studies have documented empirically the positive correlation between volume and absolute price changes or price variance over time [see, e.g., Karpoff (1987), Jain and Joh (1988), Schwert (1989), and Gallant et al. (1992)]. Jones et al. (1991) show that this relationship reflects primarily a relation between volatility of returns and the *number* of transactions. This last result is closer to our Theorem 1, since in our model only the number of transactions is endogenous, and the size of each transaction is normalized to unity.[22]

To understand Theorem 1, recall that trading volume is positive only when the two groups of speculators "switch sides" and that this occurs only when the cumulative signal changes sign. We argue that for any given current price or, equivalently, current cumulative signal, M, the absolute change in π_H (and therefore in price) is larger when averaged across signals resulting in volume than when averaged across

[22] Some of the empirical work [e.g., Schwert (1989)] establishes the positive covariance between volume and price or return volatility, as opposed to absolute price change. We can show numerically, but not analytically, that such a relationship holds for our model.

signals not resulting in volume. To see this, we first argue that for any current cumulative signal and any current signal that results in no volume there is a corresponding current signal that results in positive volume and a larger absolute price change. Suppose that $M > 0$. For volume to be positive, the signal must be below $-M$ (see Figure 2). Consider a signal s for which volume is zero (i.e., $s \geq -M$). First suppose that s is between $-M$ and M. From Figure 2, it is clear that the price change for any such signal is smaller than the price change for $s = -M$. Thus, the price change for $s \in [-M, M]$ is smaller than the smallest price change associated with positive volume. Second, suppose that $s > M$. The price change associated with this zero-volume signal is smaller than that associated with the corresponding positive-volume signal $-s$.

To summarize, we have shown that for any current value of $m > 0$ and any signal corresponding to zero volume, there is a signal that induces positive volume and results in a larger absolute price change. A similar argument holds for $m < 0$. The result does not quite follow from this argument, however, for two reasons. First, the distribution of signals may be such that, for zero-volume signals, large values of the signal are much more likely than small values, whereas the opposite is true for positive-volume signals. In fact, there is sufficient symmetry in the distributions assumed here to prevent this. Second, the intuition given above is for positive correlation *given any current price or cumulative signal.* The formal proof shows that the result can be extended to the unconditional covariance.[23]

In addition to the positive correlation between volume and absolute price changes, a positive correlation has also been observed between volume and absolute changes in mean earnings forecasts. More specifically, Ziebart (1990) documents a positive correlation between volume and the absolute percentage change, as a result of earnings announcements during the year, in the mean of analysts' earnings forecasts for the coming year. We view these annual forecasts as predictions of (more or less) long-run performance of the firm. In terms of our model, earnings announcements correspond to our signal, s_t, and analysts' forecasts correspond to the speculators' forecasts of final returns. A speculator's forecast is measured by his conditional expectation of final payoff, $\tilde{R}$. From equation (3), group j's forecast at date t of the final payoff given cumulative signal m is $\pi_H^j(m)(H - L) + L$. The mean forecast at date t is therefore

$$\mu(m) \equiv (0.5)[\pi_H^1(m) + \pi_H^2(m)](H - L) + L.$$

The absolute change in mean forecast from t to $t + 1$ given cumulative

[23] We are grateful to the referee for suggesting a method of proving this extension.

signal m and current signal s is

$$\Delta\mu(m, s) = |\mu(m + s) - \mu(m)|$$
$$= \frac{H - L}{2} |\pi_H^1(m + s) - \pi_H^1(m)$$
$$+ \pi_H^2(m + s) - \pi_H^2(m)|. \quad (4)$$

From the equation, it is clear that revisions in forecasts are related to changes in the speculator's beliefs about the probability of the high payoff, π_H^i. Since price changes are also related to changes in speculator's beliefs in our model, it is not surprising that the price change result can be extended to forecasts. In particular, we show the following result.

Theorem 2. *Absolute changes in the mean forecast of the final payoff and speculative volume are positively correlated.*

Note that no causality is implied by this relationship between volume and forecasts. Both variables are driven by a third, exogenous factor, namely the signal.

Our next result shows that our model can be consistent with the well-documented empirical observation that consecutive transaction price changes exhibit negative serial correlation [see Patell and Wolfson (1984) and the references cited there]. There are two senses in which consecutive price changes are negatively correlated in our model: transaction-to-transaction and period-to-period. First, the price changes accompanying consecutive *transactions* are of opposite sign. Second, if the price-taking group of speculators overestimates the quality of the signal, then the true expected price change in the current period is negative (positive) whenever the price change in the previous period was positive (negative). Of course, if the price-taking group underestimates the quality of the signal, the conclusion is reversed. The result is stated formally in

Theorem 3. *(a) The price changes accompanying consecutive transactions are of opposite sign. (b) Suppose that the price-taking group of speculators overestimates the quality of the signal (i.e., $\theta < \theta_o$). If the previous price change is positive (negative), then the price will decline (rise) next period, on average. On the other hand, if $\theta > \theta_o$, positive (negative) price changes will be followed on average by price increases (decreases).*

The intuition for these results is quite simple. Part (a) follows from the fact that transactions occur only when the cumulative signal crosses zero. Thus, in consecutive transactions, m crosses zero in the opposite direction, resulting in price changes of the opposite sign. For part (b), if the previous price change is positive (i.e., favorable information was revealed last period), then it is more likely that the cumulative signal is positive. If the price-taking group overestimates the signal quality, then, when the cumulative signal is positive, this group's view of the future signals is rosier than reality. That is, the true information content of the cumulative signal is less favorable than they think. Consequently, this group's expectation of next period's price is higher than its actual average value. Since the price this period is given by the price-taking group's current expectation of next period's price, the current price is higher than the true expected value of next period's price. Thus, the price will decline next period, on average. The entire argument is reversed when the previous price change is negative or when the price-taking group underestimates the quality of the signal.

Having discussed price dynamics, we now turn to results concerning the evolution of volume. Our first such result is that volume is positively autocorrelated; that is, high-volume periods tend to be followed by high-volume periods, and similarly for low-volume periods. Recall that positive volume occurs in our model only when the two groups of speculators switch sides. If such a switch occurred last period, this makes it more likely that the two groups have similar opinions this period. This, in turn, makes it more likely that opinions will again reverse and trade will occur when the current news is announced. We state the result formally as Theorem 4.

Theorem 4. *Volume is positively autocorrelated.*

This result was also obtained by Foster and Viswanathan (1993a) and Wang (1992) and is consistent with the evidence presented in Harris (1987). He estimates a median correlation coefficient of 0.586 between the number of transactions in a day and the number in the previous day. He also finds an autocorrelation in daily volume of 0.347.

Our second result concerning trading volume is, roughly speaking, that if more information is released at a given time volume tends to be larger. We model an increase in information as the appearance of two signals in a given period instead of just one and show that volume is larger on average after such an occurrence than after the release of a single signal. When two signals are released, the range or variance of possible news is increased. This makes it more likely that speculators will switch sides following such an event. Formally, we state this as Theorem 5.

Theorem 5. *Average volume when two signals are released between feasible trading dates is larger than when only one signal is released.*

Theorem 5 has several empirical applications. For example, the result implies that volume should be larger than usual on average at the opening of the market after any period in which it is closed, provided that information is released at the same rate over the closure period as over other periods. Jain and Joh (1988) and Foster and Viswanathan (1993b) show, consistent with Theorem 5, that volume is larger on average in the first hour the market is open than in other hours. Amihud and Mendelson (1987) find that volume on the opening transactions of the Dow Jones stocks constitutes, on average, 5.6 percent of the total daily volume in these stocks and is, on average, 8.4 times greater than the average volume on the closing transactions. For the Tokyo Stock Exchange, Amihud and Mendelson (1991) find that opening volume at the morning and afternoon sessions constitutes on average 11.6 percent and 9.2 percent, respectively, of the total daily trading volume. There is also some evidence that trading volume among individuals is indeed larger on Mondays, although volume among institutions is smaller [see Lakonishok and Maberly (1990)]. Osborne (1962) speculates that the difference in trading behavior occurs because institutions must wait until Monday to make trading decisions, whereas individuals make these decisions over the weekend. Another application of Theorem 5 involves the behavior of volume around simultaneous announcements by several firms in an industry. In particular, if two firms have correlated returns, then an announcement of earnings, or other data, by either has information about the other. Consequently, the volume of trade in the stock of a given firm should be higher on days in which at least one other similar firm (e.g., in the same industry) makes an announcement than on days when only the given firm does.

3. Risk-Averse Investors

One problem with the model analyzed in the previous two sections is that, due to risk neutrality of investors, short-sale restrictions are required and trading is always all-or-nothing. In this section, we examine a model in which investors are risk averse and short sales are freely allowed. Our purpose is to see to what extent our previous results are robust to this modification.

Introducing risk aversion adds considerable complexity to the problem. To restore a measure of tractability, we make three simplifying assumptions. First, we assume that investors have identical exponential (constant absolute risk aversion) utility functions. Second, we assume that investors are myopic in the sense that, in each period, they chose portfolios as if there will be no further trading. Obviously, this is a very strong assumption and is counter to the analysis of the previous sections. These two assumptions allow us to derive demand functions that are simple, independent of the investor's wealth at each date, and depend on time only through the cumulative signal. Third, we assume the asset is in zero net supply; that is, we consider only prices and volume of pure bets.

With the above assumptions, an investor in group j with current wealth W and cumulative signal m, who takes the current price of the risky asset, p, as given, chooses quantities q of the risky asset and b of the riskless asset to solve the following problem:

$$\max_{q,b} E^j[-e^{-r(\tilde{R}q+b)} \mid m]$$

$$\text{subject to} \qquad pq + b = W.$$

In this problem, E^j is the expectation using group j's model, and r is the common coefficient of absolute risk aversion of all investors. The resulting demand for the risky asset by group j is then

$$Q_j(p) = \frac{1}{r(H-L)} \ln\left[\frac{\pi_H^j(m)}{\pi_L^j(m)} \frac{H-p}{p-L}\right].$$

Equating total demand with zero net supply of the risky asset results in an equilibrium price given by (3), except that the price-taking group's θ in (2) is replaced by $\bar{\theta} = (\theta_1\theta_2)^{1/2}$. Thus, group 1's equilibrium holding of the risky asset in any period in which the cumulative signal is m is

$$Q_1^*(m) = \rho m/[2r(H-L)],$$

where $\rho = \ln(\theta_2/\theta_1)$. The volume of trade at date t is simply the absolute change in group 1's holding of the risky asset between t and $t+1$. The previous equation implies that this volume is proportional to the absolute value of the signal, s_t, at date t.

Using the foregoing analysis, it is possible to show that most of our earlier results for the risk-neutral case extend to this case as well. Although we were not able to show analytically that volume and absolute price changes are positively correlated (Theorem 1), we did verify this result numerically. Given the positive covariance of absolute price changes and volume, our result that absolute changes in the mean of the forecasts and volume are positively correlated (Theorem 2) goes through. Since volume is positive after every signal in the risk-averse model, there is no counterpart to Theorem 3(a).[24]

[24] In the risk-averse model, trade occurs even when the relative optimism of the two groups of traders does not reverse; that is, trade will occur even as the optimistic group becomes more optimistic. Thus, there is no simple relation between the sequence of transactions and price changes.

Theorem 3(b) on the serial correlation of price changes, however, goes through as before, with θ replaced by $\bar{\theta}$. Expected volume in the risk-averse model is independent of the cumulative signal and hence does not exhibit serial correlation as in the risk-neutral case (Theorem 4). Finally, it can be shown that volume following two signals is larger on average than the volume after only one signal (Theorem 5). Thus, all but one of our previous results extends to the simplified risk-aversion model.

4. Conclusions

We provide a model of speculative trading volume and price dynamics. Trading is generated by differences of opinion among traders regarding the value of the asset being traded. These differences of opinion result from different interpretations of public information announcements. Although traders are rational in our model, some view others as being irrational. Given this lack of the "common knowledge of rationality," all behavior in the model is maximizing. This article attempts to show that the approach taken here can help to explain the observed behavior of speculative markets.

We show that absolute price changes and volume are positively correlated, absolute changes in the mean forecast of the final payoff and volume are positively related, consecutive price changes exhibit negative serial correlation, volume is positively autocorrelated, and volume is larger than usual on average at the opening of the market after any period in which it is closed. Some of these results explain stylized facts documented in the empirical literature; others remain to be tested. Although the results were proved using specific functional forms, we believe them to be robust. Our implications follow from two basic properties of the model. First, the posterior beliefs of a speculator are most sensitive to the next signal when his or her current beliefs are most diffuse. Second, speculative trading is most likely when the beliefs of the traders are most diffuse. Neither of these properties is unique to the particular functional forms chosen. Moreover, the first feature is a general characteristic of Bayesian updating. The second should be true of any model of speculative trading based on differences of opinion.

An alternative approach to modeling differences of opinion is to assume that all speculators share the same likelihood function but have different prior beliefs about the final payoff, $\tilde{R}$. This approach, however, will not generate sequential trading if priors are only trivially different in the following sense. Suppose that traders agree on the set of possible "states of nature" and their probabilities but disagree about the implication of a given state of nature for the final payoff.

For example, both groups may agree that future interest rates can be either high or low, and they agree on the probabilities but disagree about the impact of high (or low) interest rates on the final payoff. Formally, suppose that all speculators agree on the prior distribution $g(\omega)$ of the state of nature ω and on the conditional distribution of the signal given the state (the likelihood function) but disagree about the mapping $R = f_j(\omega)$ relating the state to the final payoff. Thus, speculators disagree about the prior distribution of the final payoff, but j's prior beliefs are of the form $g_j(R) = g(f_j^{-1}(R))$ for all j. In this case speculators will not trade except at the last date. To see this, consider the last date. The traders will have different reservation prices based on different f_j functions. These reservation prices will result in an equilibrium price that is a function only of the past signals. At the second-to-last date, all speculators agree about the distribution of the next signal since they agree on the state probabilities and the likelihood function. Therefore speculators agree about the distribution of the next price since it depends only on current information and the distribution of the next signal. Since the only reason for trading at the second-to-last date is to speculate on the equilibrium price at the last date, speculators' agreement about the distribution of the price at the last date implies that they have no reason to trade at the seocnd-to-last date. This argument can be extended to any previous date by backward induction. Thus, to obtain sequential trading over time, one must assume that speculators disagree either about likelihoods or about the fundamental states of nature.

Appendix

We start by stating several important formulas that are used later. First, it follows immediately from the definition of $\pi_H^j(m)$ that

$$\pi_H^j(m) = \frac{a_j^m}{a_j^m + b_j^m} \qquad \text{and} \qquad \pi_L^j(m) = \pi_H^j(-m) = \frac{b_j^m}{a_j^m + b_j^m}. \tag{A1}$$

Second, the true density of the current signal, given cumulative signal m, is

$$\tau(s \mid m) = \sigma_H^0(s)\pi_H^0(m) + \sigma_L^0(s)\pi_L^0(m) = \frac{\sigma_H^0(s)\pi_H^0(m)}{\pi_H^0(m+s)}. \tag{A2}$$

It follows that

$$\begin{aligned} &\tau(s \mid -m) = \tau(-s \mid m), \\ &\tau(s \mid m) + \tau(-s \mid m) && \text{is independent of } m, \\ &\tau(-s \mid m) > \tau(-s \mid m') && \forall\ m' > m > 0, \quad s > 0, \\ &\tau(s \mid m) > \tau(-s \mid m) && \forall\ m > 0, \quad s > 0. \end{aligned} \tag{A3}$$

Third, the cumulative distribution function (cdf) corresponding to τ is

$$T(s \mid m) = \begin{cases} k_0\left[\dfrac{a_0^{-s}\pi_L^0(m)}{\alpha} + \dfrac{b_0^{-s}\pi_H^0(m)}{\beta}\right] & \text{for } s \le 0, \\ 1 - k_0\left[\dfrac{a_0^{s}\pi_H^0(m)}{\alpha} + \dfrac{b_0^{s}\pi_L^0(m)}{\beta}\right] & \text{for } s > 0. \end{cases} \tag{A4}$$

In this expression, $\alpha = -\ln(a_0)$, $\beta = -\ln(b_0)$, and

$$\frac{1}{k_0} = \int_0^\infty (a_0^s + b_0^s)\, ds = \frac{1}{\alpha} + \frac{1}{\beta}. \tag{A5}$$

Fourth, let $v(m, s)$ be volume at $t + 1$ if $m_t = m$ and $s_t = s$—1 if m and $m + s$ are of opposite sign, and 0 otherwise. Then the probability of positive volume at $t + 1$ (which equals the expected volume at $t + 1$) given cumulative signal $m_t = m$ at t is

$$\bar{v}(m) = \Pr(v_{t+1} > 0 \mid m_t = m) = E^0(v_{t+1} \mid m_t = m)$$
$$= \pi_H^0(m)\begin{cases} a_0^{-m} & \text{for } m \le 0, \\ b_0^{m} & \text{for } m > 0. \end{cases} \tag{A6}$$

Here, E^0 denotes the expectation with respect to the true distribution (of s given m in this case).

We are now ready to prove several preliminary results. Let $\lambda_t(m)$ be the unconditional density of the cumulative signal at date t. We next show that λ_t is symmetric with respect to $m = 0$.

Lemma 1. *$\lambda_t(m) = \lambda_t(-m)$ for all m and t.*

Proof. The proof is by induction on t. For $t = 1$, $\lambda_t(m) = \tau(m \mid 0) = \tau(-m \mid 0)$ since $\pi_H^0(0) = \pi_L^0(0) = \frac{1}{2}$. Now suppose $\lambda_t(m) = \lambda_t(-m)$ for all $t < n$. Then

$$\lambda_n(m) = \int_{-\infty}^{\infty} \lambda_n(m \mid m_{n-1} = x)\lambda_{n-1}(x)\, dx$$
$$= \int_{-\infty}^{\infty} \tau(m - x \mid x)\lambda_{n-1}(x)\, dx$$
$$= \int_{-\infty}^{\infty} \tau(m + x \mid -x)\lambda_{n-1}(-x)\, dx$$
$$= \int_{-\infty}^{\infty} \tau(-m - x \mid x)\lambda_{n-1}(x)\, dx$$

$$= \int_{-\infty}^{\infty} \lambda_n(-m \mid m_{n-1} = x)\lambda_{n-1}(x)\, dx = \lambda_n(-m). \tag{A7}$$

Here we have used the induction hypothesis and the fact that $\tau(s \mid -m) = \tau(-s \mid m)$. Q.E.D.

Lemma 2. *Assume group 1 is the price-taking group. Then an equilibrium price sequence is*

$$p_t(m_t) = p^1(m_t), \qquad \text{for } t = 1,\ldots,T, \tag{A8}$$

where $p^j(m_t) = H\pi^j_H(m_t) + L\pi^j_L(m_t)$. Also, for every t, m, and j, the reservation price for the risky asset of group j at date t after cumulative signal m is given by $r^j_t(m) = E^j[p_{t+1}(m + \tilde{s}) \mid m]$, where E^j denotes the expectation with respect to group j's posterior beliefs. Thus, $r^1(m) = p^1(m)$ and $r^2(m) = E^2[p^1(m + \tilde{s}) \mid m]$. Obviously, if group 2 is the price-taking group, the lemma goes through with all group indices reversed.

Proof. The proof uses a backward induction argument. Consider date T and suppose the cumulative signal at date T is m_T. Since this is the last period, group j values the risky asset at $p^j(m_T)$. Consequently, if $p^2(m_T) > p^1(m_T)$, group 2 will offer to buy as much as group 1 wants to sell at price $p^1(m_T)$. If $p^2(m_T) \le p^1(m_T)$, group 2 will offer to sell as much as it has at price $p^1(m_T)$. Thus, the equilibrium price at date T is $p^1(m_T)$.

Now consider date $T - 1$. If group 1 purchases d units of the risky asset at price p, its expected utility from this purchase is

$$d[E^1(p^1(m_{T-1} + \tilde{s}) \mid m_{T-1}) - p] = d[p^1(m_{T-1}) - p],$$

since $E^j[\pi^j(m + \tilde{s}) \mid m] = \pi^j(m)$. Thus, at $T - 1$, group 1's reservation price is $p^1(m_{T-1})$. Similarly, group 2 investors know that they will be able to buy or sell the risky asset at date T at price $p^1(m_{T-1} + \tilde{s})$, so their reservation price at date $T - 1$ is $E^2(p^1(m_{T-1} + \tilde{s}) \mid m_{T-1})$. Consequently, by the same argument as before, the equilibrium price at date $T - 1$ is $p^1(m_{T-1})$.

Obviously, the argument can be repeated for $t = T - 2,\ldots,1$. Q.E.D.

Hereafter, we denote the price-taking group's π^j_R, θ_j, and p^j by π_R, θ, and p, respectively. Let

$$\Delta\pi(m, s) = |\pi_H(m + s) - \pi_H(m)|.$$

Then

$$|\Delta p_t| = |p(m_{t+1}) - p(m_t)| = (H - L)\Delta\pi(m_t, s_t). \tag{A9}$$

It is easy to check that

$$\Delta\pi(m, s) < \Delta\pi(m, -s) \qquad \forall\ m > 0, \quad s > 0, \tag{A10}$$

$$\Delta\pi(m, s) \text{ is monotone decreasing in } m \qquad \forall\ m > 0, \quad s > 0.$$

Now, let

$$q_i(m) = E^0[\Delta\pi(m, \tilde{s}) \mid m_t = m, v_{t+1} = i], \qquad \text{for } i = 0,1,$$

$$q(m) = E^0[\Delta\pi(m, \tilde{s}) \mid m_t = m].$$

Note that q_i is independent of t.

Lemma 3. *Suppose $\theta_0 > \theta$. For $m > 0$ ($m < 0$), $E^0[p_{t+1} \mid m_t = m] <$ ($>$) p_t [i.e., if the current price exceeds (is exceeded by) the unconditional average price, $p(0)$], then next period's price will be below (above) this period's on average.*

Proof. This will be shown only for $m > 0$. The proof for $m < 0$ is symmetric. Since

$$E^0[p_{t+1} - p_t \mid m_t = m] = (H - L)E^0[\pi_H(m + \tilde{s}) - \pi_H(m) \mid m],$$

it suffices to show that $E^0[\pi_H(m + \tilde{s}) \mid m] < \pi_H(m)$ for all $m > 0$. From (A2),

$$\begin{aligned}
&E^0[\pi_H(m, \tilde{s}) \mid m] \\
&= \int_{-\infty}^{\infty} \pi_H(m + s)\tau(s \mid m)\, ds \\
&= \pi_H^0(m) \int_{-\infty}^{\infty} \frac{\pi_H(m + s)\sigma_H^0(s)}{\pi_H^0(m + s)}\, ds \\
&= k_0\pi_H^0(m)\left[\int_0^{\infty} \frac{\pi_H(m + s)a_0^s}{\pi_H^0(m + s)}\, ds + \int_{-\infty}^{0} \frac{\pi_H(m + s)b_0^{-s}}{\pi_H^0(m + s)}\, ds\right] \\
&= k_0\pi_H^0(m) \int_0^{\infty} \left[\frac{\pi_H(m + s)a_0^s}{\pi_H^0(m + s)} + \frac{\pi_H(m - s)b_0^s}{\pi_H^0(m - s)}\right] ds.
\end{aligned} \tag{A11}$$

Suppose that we show

$$\pi_H^0(m)\left[\frac{\pi_H(m + s)a_0^s}{\pi_H^0(m + s)} + \frac{\pi_H(m - s)b_0^s}{\pi_H^0(m - s)}\right] < \pi_H(m)(a_0^s + b_0^s), \qquad \text{for all } s > 0. \tag{A12}$$

Then integrating both sides and using (A5) gives the result. Therefore, it suffices to establish (A12), which can be rewritten as

$$\pi_H(m+s)\frac{\pi_H^0(s)\pi_H^0(m)}{\pi_H^0(m+s)} + \pi_H(m-s)\frac{\pi_H^0(-s)\pi_H^0(m)}{\pi_H^0(m-s)} < \pi_H(m), \tag{A13}$$

by dividing by $a_0^s + b_0^s$ and using (A1). To show (A13), note that it is satisfied as an equality at $\theta_0 = \theta$. Therefore, it suffices to show that the left-hand side of (A13) is strictly decreasing in θ_0 for $m > 0$. But it is easy to check that $z(\theta_0) \equiv \pi_H^0(s)\pi_H^0(m)/\pi_H^0(m+s)$ is decreasing in θ_0 for $m > 0$ and $s > 0$. Also, $\pi_H(m+s) > \pi_H(m-s)$ for $s > 0$, and

$$\frac{\pi_H^0(s)\pi_H^0(m)}{\pi_H^0(m+s)} + \frac{\pi_H^0(-s)\pi_H^0(m)}{\pi_H^0(m-s)} = 1. \tag{A14}$$

Therefore, denoting the left-hand side of (A13) by $Z(\theta_0)$, we have (since π_H is independent of θ_0),

$$Z'(\theta_0) = z'(\theta_0)[\pi_H(m+s) - \pi_H(m-s)] < 0. \qquad \text{Q.E.D.}$$

Lemma 4. *Suppose g and h are monotone functions on $\Re$ with either both increasing or both decreasing and with strict monotonicity on a set of positive Lebesgue measure. Let $\tilde{X}$ be an absolutely continuous random variable. Then $Cov[g(\tilde{X}), h(\tilde{X})] > 0$.*

Proof. We can assume, without loss of generality, that $Eg(\tilde{X}) = 0$. We also consider only the case in which g and h are increasing, since the other case is symmetric. Then there exists an x_0 such that $g(x_0) = 0$. Now $g(x) > 0$ implies that $x > x_0$ which in turn implies $h(x) \geq h(x_0)$. Similarly, $g(x) < 0$ implies that $h(x) \leq h(x_0)$. Moreover, at least one of the inequalities on h must be strict over a set of values of x with positive probability. Therefore, $g(x)h(x) \geq g(x)h(x_0)$ with strict inequality for a set of positive probability. Consequently, $\text{Cov}[g(\tilde{X}), h(\tilde{X})] = E[g(\tilde{X})h(\tilde{X})] > h(x_0)Eg(\tilde{X}) = 0$. Q.E.D.

Lemma 5. *Suppose g and h satisfy the hypotheses of Lemma 4 for $x \geq 0$, and g, h, and the density f of $\tilde{X}$ are symmetric with respect to $x = 0$. Then $Cov[g(\tilde{X}), h(\tilde{X})] > 0$.*

Proof. By symmetry, g and h have the same monotonicity for $x \leq 0$. Now

$$\begin{aligned} E[g(\tilde{X})h(\tilde{X})] &= F(0)E[g(\tilde{X})h(\tilde{X}) \mid \tilde{X} \leq 0] \\ &\quad + [1 - F(0)]E[g(\tilde{X})h(\tilde{X}) \mid \tilde{X} \geq 0] \\ &> F(0)E[g(\tilde{X}) \mid \tilde{X} \leq 0]E[h(\tilde{X}) \mid \tilde{X} \leq 0] \end{aligned}$$

$$+ [1 - F(0)]E[g(\tilde{X}) \mid \tilde{X} \geq 0]E[h(\tilde{X}) \mid \tilde{X} \geq 0]$$

by Lemma 4 applied to each conditional expectation. Now, by symmetry, it is easy to show that $E[g(\tilde{X}) \mid \tilde{X} \leq 0] = E[g(\tilde{X}) \mid \tilde{X} \geq 0] = E[g(\tilde{X})]$. The result now follows from the preceding inequality.
Q.E.D.

***Lemma* 6.** *Suppose $g(m, s)$ and $h(m, s)$ are two functions with the following properties: $\bar{g}(m) \equiv E^o[g(m, \tilde{s}) \mid m]$ and $\bar{h}(m) = E^o[h(m, \tilde{s}) \mid m]$ are symmetric about $m = 0$, both strictly monotone in the same direction for $m > 0$, and $Cov[g(m, \tilde{s}), h(m, \tilde{s}) \mid m] \geq 0$ for all m with strict inequality for $m \neq 0$. Then $Cov[g(\tilde{m}, \tilde{s}), h(\tilde{m}, \tilde{s})] > 0$.*

Proof. Let $\bar{g} = E^0\bar{g}(\tilde{m})$, $\bar{h} = E^0\bar{h}(\tilde{m})$. Then it is easy to check that

$$\begin{aligned}\text{Cov}[g(\tilde{m}, \tilde{s}), h(\tilde{m}, \tilde{s})] = {} & E^0[g(\tilde{m}, \tilde{s})(h(\tilde{m}, \tilde{s}) - \bar{h}(\tilde{m}))] \\ & + E^0[g(\tilde{m}, \tilde{s})(\bar{h}(\tilde{m}) - \bar{h})].\end{aligned}$$

But the first term is the expectation over $\tilde{m}$ of the covariance conditional on $\tilde{m}$. Since the conditional covariance is assumed to be strictly positive everywhere (except at zero), the first term is positive. The second term is simply $\text{Cov}[\bar{g}(\tilde{m}), \bar{h}(\tilde{m})]$. That this is positive follows from Lemma 5 and the symmetry of λ_t (Lemma 1). Q.E.D.

***Lemma* 7.** *$q_1(m) \geq q_0(m)$ for all m with strict inequality for $m \neq 0$.*

Proof. It is easy to check that $q_1(0) = q_0(0)$. We provide a detailed proof of strict inequality only for $m < 0$. The proof for $m > 0$ is symmetric.

First, $v_{t+1} > 0$ if and only if $s_t > -m_t$. Using (A9), it suffices to show that

$$\int_{-m}^{\infty} \Delta\pi_H(m, s)\frac{\tau(s \mid m)\, ds}{1 - T(-m \mid m)} > \int_{-\infty}^{-m} \Delta\pi_H(m, s)\frac{\tau(s \mid m)\, ds}{T(-m \mid m)}. \tag{A15}$$

It is easy to check that

$$|\pi_H(m + s) - \pi_H(m)| < |\pi_H(0) - \pi_H(m)|, \quad \text{for all } m < s < -m,$$

and

$$|\pi_H(m + s) - \pi_H(m)| < |\pi_H(m - s) - \pi_H(m)|, \quad \text{for all } s \leq m.$$

Therefore,

$$\int_{-\infty}^{-m} \Delta\pi_H(m, s)\frac{\tau(s \mid m)\, ds}{T(-m \mid m)}$$

$$\leq \int_{-\infty}^{m} |\pi_H(m - s) - \pi_H(m)| \frac{\tau(s \mid m)\, ds}{T(-m \mid m)}$$

$$+ |\pi_H(0) - \pi_H(m)| \frac{T(-m \mid m) - T(m \mid m)}{T(-m \mid m)}$$

$$= \int_{-m}^{\infty} |\pi_H(m + s) - \pi_H(m)| \frac{\tau(-s \mid m)\, ds}{T(-m \mid m)}$$

$$+ |\pi_H(0) - \pi_H(m)| \frac{T(-m \mid m) - T(m \mid m)}{T(-m \mid m)}, \qquad \text{(A16)}$$

using a change of variable in the integral. Comparing this to the left-hand side of (A15), we see that, on each side, we are taking the expectation of an increasing function of s, namely $|\pi_H(m + s) - \pi_H(m)|$. The cdf on the left-hand side is simply

$$F(x) \equiv \frac{T(x \mid m) - T(-m \mid m)}{1 - T(-m \mid m)} \qquad \text{for } x \geq -m. \qquad \text{(A17)}$$

The cdf on the right-hand side, $G(x)$, has a mass point of

$$G_0 = \frac{T(-m \mid m) - T(m \mid m)}{T(-m \mid m)} \qquad \text{(A18)}$$

at $x = -m$, and

$$G(x) = G_0 + \int_{-m}^{x} \frac{\tau(-s \mid m)\, ds}{T(-m \mid m)} = G_0 + \int_{-x}^{m} \frac{\tau(s \mid m)\, ds}{T(-m \mid m)}$$

$$= \frac{T(-m \mid m) - T(-x \mid m)}{T(-m \mid m)}, \qquad \text{for } x > -m. \qquad \text{(A19)}$$

It is easy to check that G is in fact a cdf.

To establish (A15), is suffices to show that F stochastically dominates G in the first-order sense; that is,

$$F(x) < G(x) \qquad \text{for } -m \leq x < \infty.$$

Rearranging terms, we see that this is equivalent to

$$T(-m \mid m)[1 - T(x \mid m)] > T(-x \mid m)[1 - T(-m \mid m)].$$

Substituting from (A4) into the preceding inequality and rearranging shows that this inequality is equivalent to

$$a_0^x(a_0^m - 1)/\alpha + b_0^x(b_0^m - 1)/\beta > 0. \qquad \text{(A20)}$$

This is true, however, since a_0 and b_0 are less than 1 and $m < 0$.

Q.E.D.

Lemma 8. *$Cov[v_t, \Delta\pi_t | m_t = m] \geq 0$ for all m with strict inequality for $m \neq 0$.*

Proof.

$$\begin{aligned}\text{Cov}[v_t, \Delta\pi_t \mid m_t = m] &= E^0[v(m, \tilde{s})\Delta\pi(m, \tilde{s}) \mid m] - \bar{v}(m)q(m) \\ &= \bar{v}(m)[q_1(m) - q(m)] \\ &\geq 0 \qquad \text{with } > \text{ for } m \neq 0,\end{aligned}$$

by Lemma 7, since $q(m)$ is a weighted average of $q_1(m)$ and $q_0(m)$.

Q.E.D.

Theorem 1. *$Cov[v(\tilde{m}, \tilde{s}), \Delta\pi(\tilde{m}, \tilde{s})] > 0$.*

Proof. It easy to check from (A6) that $\bar{v}(m) = \bar{v}(-m)$ and that $\bar{v}$ is strictly decreasing in m for $m \geq 0$. Similarly, that $q(m) = q(-m)$ follows from (A23). To apply Lemma 6, we must show that, for $m' > m > 0$, the following expression is positive:

$$\begin{aligned}q(m) - q(m') &= \int_{-\infty}^{\infty} [\Delta\pi(m, s) - \Delta\pi(m', s)]\tau(s \mid m)\, ds \\ &\quad + \int_{-\infty}^{\infty} [\Delta\pi(m', s)[\tau(s \mid m) - \tau(s \mid m')]\, ds. \qquad \text{(A21)}\end{aligned}$$

In fact, we can show that each integral in this expression is positive:

$$\begin{aligned}&\int_{-\infty}^{\infty} \Delta\pi(m', s)[\tau(s \mid m) - \tau(s \mid m')]\, ds \\ &= \int_{0}^{\infty} \Delta\pi(m', s)[\tau(s \mid m) - \tau(s \mid m')]\, ds \\ &\quad + \int_{0}^{\infty} \Delta\pi(m', -s)[\tau(-s \mid m) - \tau(-s \mid m')]\, ds \\ &> \int_{0}^{\infty} \Delta\pi(m', -s)[\tau(s \mid m) - \tau(s \mid m')]\, ds \\ &\quad + \int_{0}^{\infty} \Delta\pi(m', s)[\tau(-s \mid m) - \tau(-s \mid m')]\, ds \\ &= 0,\end{aligned}$$

where the inequality follows from (A3) and (A10), and the final equality follows from (A3).

Now let $d(m, s) = \pi_H(m + s) - \pi_H(m)$. Then

$$\int_{-\infty}^{\infty} [\Delta\pi(m, s) - \Delta\pi(m', s)]\tau(s \mid m)\, ds$$

$$= \int_0^{\infty} [d(m, s) - d(m', s)]\tau(s \mid m)\, ds$$

$$+ \int_0^{\infty} [d(m', -s) - d(m, -s)]\tau(-s \mid m)\, ds$$

$$> \int_0^{\infty} [d(m, s) - d(m', s) + d(m', -s) - d(m, -s)]\tau(-s \mid m)\, ds,$$

where the equality follows from a change of variable and the inequality uses (A3) and (A10). Therefore, it suffices to show that $d(m, s) - d(m, -s)$ is decreasing in m for s and $m > 0$. This is equivalent to showing that $\pi_H(m + s) - \pi_H(m - s)$ is decreasing in m for s and $m > 0$, or, since

$$\pi_H(x) = (-\ln\theta)\pi_H(x)[1 - \pi_H(x)],$$

that

$$\pi_H(m + s)[1 - \pi_H(m + s)] < \pi_H(m - s)[1 - \pi_H(m - s)].$$

This last inequality is easy to check.

This shows that the first integral of (A21) is positive and completes the proof that q is decreasing in m for $m > 0$. The theorem now follows from Lemmas 6 and 7. Q.E.D.

Lemma 9. *$E^o[\Delta\mu(m, \tilde{s}) \mid m_t = m, v_{t+1} > 0] \geq E^o[\Delta\mu(m, \tilde{s}) \mid m_t = m, v_{t+1} = 0]$, for all t and m, with $>$ for $m \neq 0$.*

Proof. Using (4) and the fact that $\pi_H^1(m + s) - \pi_H^1(m) < 0$ if and only if $\pi_H^2(m + s) - \pi_H^2(m) < 0$,

$$\Delta\mu(m, s) = \frac{H - L}{2}[|\pi_H^1(m + s) - \pi_H^1(m)| + |\pi_H^2(m + s) - \pi_H^2(m)|]. \tag{A22}$$

Therefore, it suffices to show

$$E^0[|\pi_H^j(m + \tilde{s}) - \pi_H^j(m)| \mid m_t = m, v_{t+1} > 0] > E^0[|\pi_H^j(m + \tilde{s}) - \pi_H^j(m)| \mid m_t = m, v_{t+1} = 0],$$

for $j = 1,2$. But this is exactly (A15) with π_H^j instead of π_H. The proof of Theorem 1 goes through without modification. Q.E.D.

Lemma 10. *$Cov[\Delta\mu(m, \tilde{s}), \Delta\pi(m, \tilde{s}) \mid m_t = m] \geq 0$ for all m with strict inequality for $m \neq 0$.*

Proof. Using (A22), the proof of Lemma 8 goes through. Q.E.D.

Theorem 2. *$Cov[\Delta\mu(\tilde{m}, \tilde{s}), \Delta\pi(\tilde{m}, \tilde{s})] > 0$.*

Proof. The same proof as for Theorem 1 (using Lemma 10 instead of Lemma 8) goes through if we show that $E^0[\Delta\mu(m, \tilde{s}) \mid m]$ is symmetric with respect to $m = 0$ and decreasing for $m > 0$. This follows from the proof for q in Theorem 1 and (A22). Q.E.D.

Theorem 3. *(a) The price changes accompanying consecutive transactions are of opposite sign.*

(b) If $\theta_0 > \theta$, then $E^0[\Delta p_t \mid \Delta p_{t-1} > 0] < 0$ and $E^0[\Delta p_t \mid \Delta p_{t-1} < 0] > 0$. If $\theta_0 < \theta$, then $E^0[\Delta p_t \mid \Delta p_{t-1} > 0] > 0$ and $E^0[\Delta p_t \mid \Delta p_{t-1} < 0] < 0$.

Proof. Part (a) was explained in the text. For part (b), we prove only the first inequality; the proofs of the others are symmetric. The first inequality is equivalent to $E^0[\Delta p_t \mid s_{t-1} > 0] < 0$. To prove this inequality, first recall that $\tau(s \mid -m) = \tau(-s \mid m)$, so

$$\begin{aligned} &E^0[\Delta p_t \mid -m] \\ &\quad = (H - L)E^0[\pi_H(-m + s) - \pi_H(-m) \mid -m] \\ &\quad = (H - L)\int_{-\infty}^{\infty} [\pi_H(-m + s) - \pi_H(-m)]\tau(s \mid -m)\, ds \\ &\quad = (H - L)\int_{-\infty}^{\infty} [\pi_H(-m + s) - \pi_H(-m)]\tau(-s \mid m)\, ds \\ &\quad = (H - L)\int_{-\infty}^{\infty} [\pi_H(-m - t) - \pi_H(-m)]\tau(t \mid m)\, dt \\ &\quad = (H - L)\int_{-\infty}^{\infty} [\pi_L(m + t) - \pi_L(m)]\tau(t \mid m)\, dt \\ &\quad = -E^0[\Delta p_t \mid m]. \end{aligned} \tag{A23}$$

Now,

$$\begin{aligned} &E^0[\Delta p_t \mid s_{t-1} > 0] \\ &\quad = \int_{-\infty}^{\infty} E^0[\Delta p_t \mid m]\lambda_t(m \mid s_{t-1} > 0)\, dm \end{aligned}$$

$$= \int_0^\infty E^0[\Delta p_t \mid m]\lambda_t(m \mid s_{t-1} > 0)\ dm$$

$$+ \int_0^\infty E^0[\Delta p_t \mid -m]\lambda_t(-m \mid s_{t-1} > 0)\ dm$$

$$= \int_0^\infty E^0[\Delta p_t \mid m][\lambda_t(m \mid s_{t-1} > 0) - \lambda_t(-m \mid s_{t-1} > 0)]\, dm, \quad \text{(A24)}$$

where $\lambda_t(m \mid s_{t-1})$ is the conditional density of m_t given s_{t-1}. By Lemma 3, $E^0[\Delta p_t \mid m] < 0$ for $m > 0$, so it suffices to show that

$$\lambda_t(m \mid s_{t-1} > 0) > \lambda_t(-m \mid s_{t-1} > 0) \qquad \text{for every } m > 0 \text{ and } t > 0.$$

Consequently, it also suffices to show that

$$\lambda_t(m \mid s_{t-1}) > \lambda_t(-m \mid s_{t-1}) \qquad \text{for every } m > 0, s_{t-1} > 0, \text{ and } t > 0.$$

But, this inequality is equivalent to

$$\tau(s \mid m)\lambda_t(m) > \tau(s \mid -m)\lambda_t(-m)$$

or

$$\tau(s \mid m) > \tau(-s \mid m) \qquad \text{for every } m > 0 \text{ and } s > 0,$$

using Lemma 1. This inequality follows easily from the facts that $\sigma_H^0(s) > \sigma_L^0(s)$ for $s > 0$ and $\pi_H^0(m) > \frac{1}{2} > \pi_L^0(m)$ for $m > 0$. Q.E.D.

Theorem 4. *Cov*$[v_{t+1}, v_t] > 0$ *for all t.*

Proof. We first show that $E^0(v_{t+1} \mid v_t > 0) > E^0(v_{t+1} \mid v_t = 0)$. To show this, we begin by calculating the expectations conditional on $m_{t-1} = n$ for any n. First suppose $n > 0$. Then

$$E^0(v_{t+1} \mid v_t > 0,\ m_{t-1} = n)$$

$$= E^0(v_{t+1} \mid m_t < 0,\ m_{t-1} = n)$$

$$= \int_{-\infty}^0 E^0(v_{t+1} \mid m_t = m,\ m_{t-1} = n)\frac{\tau(m - n \mid n)\ dm}{T(-n \mid n)}$$

$$= \int_{-\infty}^0 \pi_H^0(m)\, a_0^{-m}\frac{\tau(m - n \mid n)\ dm}{T(-n \mid n)}$$

$$= \frac{\pi_H^0(n)}{T(-n \mid n)}\int_{-\infty}^0 a_0^{-m}\sigma_H^0(m - n)\ dm$$

$$= \frac{\pi_H^0(n)\, k_0\, b_0^n}{T(-n \mid n)(\alpha + \beta)}$$

$$= \frac{k_0}{(\alpha + \beta)}. \quad \text{(A25)}$$

The third equality follows from (A6), the fourth equality from (A2), the fifth from the definition of σ_H^0, and the last from (A1) and (A4). A similar calculation shows that this formula is also valid for $n \le 0$. Since the expression does not depend on n, $E^0(v_{t+1} \mid v_t > 0) = k_0/(\alpha + \beta)$.

Similarly, for $n > 0$,

$$\begin{aligned} &E^0(v_{t+1} \mid v_t = 0, m_{t-1} = n) \\ &= E^0(v_{t+1} \mid m_t > 0, m_{t-1} = n) \\ &= \int_0^\infty \pi_H^0(m) b_0^m \frac{\tau(m - n \mid n)\, dm}{1 - T(-n \mid n)} \\ &= \frac{\pi_H^0(n)}{1 - T(-n \mid n)} \int_0^\infty b_0^m \sigma_H^0(m - n)\, dm \\ &= \frac{\pi_H^0(n) k_0}{1 - T(-n \mid n)} \left[\int_0^n b_0^m b_0^{n-m}\, dm + \int_n^\infty b_0^m a_0^{m-n}\, dm \right] \\ &= \frac{\pi_H^0(n) k_0 b_0^n}{1 - T(-n \mid n)} \left[n + \frac{1}{\alpha + \beta} \right] \\ &= \frac{k_0}{\alpha + \beta} \frac{\pi_H^0(n) b_0^n}{1 - \pi_H^0(n) b_0^n} [n(\alpha + \beta) + 1]. \end{aligned} \tag{A26}$$

It can be shown that this expectation is symmetric with respect to $n = 0$; that is, $E^0(v_{t+1} \mid v_t = 0, m_{t+1} = n) = E^0(v_{t+1} \mid v_t = 0, m_{t+1} = -n)$.

It suffices to show, therefore, that

$$\frac{\pi_H^0(n) b_0^n}{1 - \pi_H^0(n) b_0^n} [n(\alpha + \beta) + 1] < 1, \qquad \text{for every } n > 0, \tag{A27}$$

or

$$a_0^{-n} + b_0^{-n} - n(\alpha + \beta) > 2. \tag{A28}$$

At $n = 0$, this is an equality, and it is easy to check that the left-hand side increases with n for $n \ge 0$.

Now

$$\begin{aligned} \mathrm{Cov}[v_{t+1}, v_t] &= E^0[v_{t+1} \mid v_t > 0]\Pr[v_t > 0] - E^0[v_{t+1}]E^0[v_t] \\ &> E^0[v_{t+1}]\Pr[v_t > 0] - E^0[v_{t+1}]E^0[v_t] = 0, \end{aligned}$$

where the inequality follows from the inequality established in the first part of the proof. Q.E.D.

We next calculate the density and cdf of the sum of two signals.

Let $\tilde{s}_1$ and $\tilde{s}_2$ be two draws from the true distribution of $\tilde{s}$ conditional on $\tilde{R} = R \in \{H, L\}$. Denote the true density of $\tilde{s}_1 + \tilde{s}_2$ by $w(\cdot \mid R)$. First, suppose that $R = H$. Then, for $t > 0$,

$$\begin{aligned} w(t \mid H) &= \int_{-\infty}^{\infty} \sigma_H^0(t - s)\sigma_H^0(s)\, ds \\ &= k_0^2\left[\int_0^t a_0^{t-s}a_0^s\, ds + \int_t^{\infty} b_0^{s-t}a_0^s\, ds + \int_{-\infty}^0 a_0^{t-s}b_0^{-s}\, ds\right] \\ &= k_0^2 a_0^t\left(\frac{2}{\alpha + \beta} + t\right). \end{aligned} \tag{A29}$$

For $t < 0$,

$$\begin{aligned} w(t \mid H) &= k_0^2\left[\int_t^0 b_0^{s-t}b_0^{-s}\, ds + \int_0^{\infty} b_0^{s-t}a_0^s\, ds \right.\\ &\qquad \left. + \int_{-\infty}^t a_0^{t-s}b_0^{-s}\, ds\right] \\ &= k_0^2 b_0^{-t}\left(\frac{2}{\alpha + \beta} - t\right). \end{aligned} \tag{A30}$$

For $R = L$, one simply reverses the roles of a_0 and b_0, since σ_L^0 is σ_H^0 with a_0 and b_0 interchanged. The formulas can be stated compactly as

$$w(t \mid R) = \begin{cases} k_0^2 a_0^{|t|}\left(\dfrac{2}{\alpha + \beta} + |t|\right) & \text{for } t \geq 0 \text{ and } R = H \\ & \quad \text{or } t \leq 0 \text{ and } R = L, \\ k_0^2 b_0^{|t|}\left(\dfrac{2}{\alpha + \beta} + |t|\right) & \text{for } t < 0 \text{ and } R = H \\ & \quad \text{or } t > 0 \text{ and } R = L. \end{cases} \tag{A31}$$

It is now easy to check, using integration by parts, that

$$W(t \mid H) = \begin{cases} 1 - \dfrac{k_0^2 a_0^t}{\alpha}\left(\dfrac{1}{\alpha} + \dfrac{2}{\alpha + \beta} + t\right) & \text{for } t \geq 0, \\ \dfrac{k_0^2 b_0^{-t}}{\beta}\left(\dfrac{1}{\beta} + \dfrac{2}{\alpha + \beta} - t\right) & \text{for } t < 0, \end{cases} \tag{A32}$$

Also, $W(t \mid L)$ is the same as $W(t \mid H)$ with a_0 and b_0 interchanged. We may now prove our next major result.

Theorem 5. *$E^o(v_{t+2} \mid$ trade is not allowed at $t + 1) > E^o(v_{t+1})$.*

Proof. We prove this conditional on $m_t = m$ for any m. The result then follows by integrating over m. First suppose $m < 0$. Then

$$E^0(v_{t+2} \mid m_t = m \text{ and trade is not allowed at } t + 1)$$
$$= \frac{\pi_H^0(m) a_0^{-m} k_0^2}{\alpha}\left(\frac{1}{\alpha} + \frac{2}{\alpha + \beta} - m\right)$$
$$+ \frac{\pi_L^0(m) b_0^{-m} k_0^2}{\beta}\left(\frac{1}{\beta} + \frac{2}{\alpha + \beta} - m\right)$$
$$= \pi_H^0(m) a_0^{-m} k_0^2\left(\frac{-m}{k_0} + \frac{1}{k_0^2}\right)$$
$$= \pi_H^0(m) a_0^{-m}(1 - mk_0). \tag{A33}$$

The first equality uses the fact that $v_{t+2} > 0$ if and only if $s_t + s_{t+1} > -m$ and (A32) to average $1 - W(-m \mid R)$ over $R = H,L$, using $\pi_H^0(m)$ and $\pi_L^0(m)$. The second equality uses (A1) and (A5). Now

$$E^0(v_{t+1} \mid m_t = m) = 1 - T(-m \mid m) = \pi_H^0(m)(a_0)^{-m}.$$

The result for $m < 0$ follows since $1 - mk_0 > 1$. The proof for $m > 0$ is symmetric. Q.E.D.

References

Admati, A., and P. Pfleiderer, 1988, "A Theory of Intraday Patterns: Volume and Price Variability," *Review of Financial Studies,* 1, 3–40.

Amihud, Y., and H. Mendelson, 1987, "Trading Mechanisms and Stock Returns: An Empirical Investigation," *Journal of Finance,* 42, 533–553.

Amihud, Y., and H. Mendelson, 1991, "Volatility, Efficiency, and Trading: Evidence from the Japanese Stock Market," *Journal of Finance,* 46, 1765–1789.

Bamber, L. S., 1987, "Unexpected Earnings, Firm Size, and Trading Volume Around Quarterly Earnings Announcements," *Accounting Review,* 62, 510–532.

Beaver, W., 1968, "The Information Content of Annual Earnings Announcements," *Empirical Research in Accounting, Journal of Accounting Research* (Suppl. to vol. 6), 67–92.

Blough, S., 1988, "Differences of Opinion and the Information Value of Prices," working paper, The Johns Hopkins University.

Blume, L., D. Easley, and M. O'Hara, 1991, "Market Statistics and Technical Analysis: The Role of Volume," working paper, Cornell University.

Brock, W. A., and A. W. Kleidon, 1992, "Periodic Market Closure and Trading Volume: A Model of Intraday Bids and Asks," *Journal of Economic Dynamics and Control,* 16, 451–489.

De Long, J. B., A. Shleifer, L. H. Summers, and R. J. Waldmann, 1990, "Noise Trader Risk in Financial Markets," *Journal of Political Economy,* 98, 703–738.

Detemple, J., and S. Murthy, 1992, "Intertemporal Asset Pricing with Heterogeneous Information and Beliefs," working paper, Columbia University.

Diamond, D. W., and R. E. Verrecchia, 1981, "Information Aggregation in a Noisy Rational Expectations Economy," *Journal of Financial Economics,* 9, 221–235.

Foster, F. D., and S. Viswanathan, 1990, "A Theory of the Interday Variations in Volume, Variance, and Trading Costs in Securities Markets," *Review of Financial Studies,* 3, 593–624.

Foster, F. D., and S. Viswanathan, 1993a, "The Effect of Public Information and Competition on Trading Volume and Price Volatility," *Review of Financial Studies,* 6, 23–56.

Foster, F. D., and S. Viswanathan, 1993b, "Variations in Trading Volumes, Return Volatility and Trading Costs: Evidence on Recent Price Formation Models," *Journal of Finance,* 48, 187–211.

Gallant, A. R., P. E. Rossi, and G. Tauchen, 1992, "Stock Prices and Volume," *Review of Financial Studies,* 5, 199–242.

Grossman, S. J., and J. E. Stiglitz, 1976, "Information and Competitive Price Systems," *American Economic Review,* 66, 246–253.

Grossman, S. J., and J. E. Stiglitz, 1980, "On the Impossibility of Informationally Efficient Markets," *American Economic Review,* 70, 393–408.

Grundy, B. D., and M. McNichols, 1989, "Trade and Revelation of Information through Prices and Direct Disclosure," *Review of Financial Studies,* 2, 495–526.

Harris, L., 1987, "Transaction Data Tests of the Mixture of Distributions Hypothesis," *Journal of Financial and Quantitative Analysis,* 22, 127–141.

Harrison, J. M., and D. M. Kreps, 1978, "Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations," *Quarterly Journal of Economics,* 93, 323–336.

Hellwig, M., 1980, "On the Aggregation of Information in Competitive Markets," *Journal of Economic Theory,* 22, 477–498.

Hindy, A., 1989, "An Equilibrium Model of Futures Markets Dynamics," working paper, MIT.

Holthausen, R. W., and R. E. Verrecchia, 1990, "The Effect of Informedness and Consensus on Price and Volume Behavior," *Accounting Review,* 65, 191–208.

Jain, P. C., and G-H. Joh, 1988, "The Dependence between Hourly Prices and Trading Volume," *Journal of Financial and Quantitative Analysis,* 23, 269–283.

Jones, C. M., G. Kaul, and M. L. Lipson, 1991, "Transactions, Volume and Volatility," working paper, University of Michigan.

Kandel, E., and N. D. Pearson, 1992, "Differential Interpretation of Public Signals and Trade in Speculative Markets," working paper, University of Rochester.

Karpoff, J. M., 1987, "The Relation between Price Changes and Trading Volume: A Survey," *Journal of Financial and Quantitative Analysis,* 22, 109–126.

Kim, O., and R. E. Verrecchia, 1991a, "Trading Volume and Price Reactions to Public Announcements," *Journal of Accounting Research,* 29, 302–321.

Kim, O., and R. E. Verrecchia, 1991b, "Market Reactions to Anticipated Announcements," *Journal of Financial Economics,* 30, 273–309.

Kyle, A., 1985, "Continuous Auctions and Insider Trading," *Econometrica,* 53, 1315–1335.

Lakonishok, J., and E. Maberly, 1990, "The Weekend Effect: Trading Patterns of Individual and Institutional Investors," *Journal of Finance,* 45, 231–243.

Lakonishok, J., and S. Smidt, 1986, "Capital Gains Taxation and Volume of Trading," *Journal of Finance,* 41, 951–974.

Lang, L. H. P., R. H. Litzenberger, and V. Madrigal, 1992, "Testing Financial Market Equilibrium under Asymmetric Information," *Journal of Political Economy,* 100, 317–348.

Milgrom, P., and N. Stokey, 1982, "Information, Trade and Common Knowledge," *Journal of Economic Theory,* 26, 17–27.

Osborne, M. F. M., 1962, "Periodic Structure in the Brownian Motion of the Stock Market," *Operations Research,* 10, 345–379.

Patell, J. M., and M. A. Wolfson, 1984, "The Intraday Speed of Adjustment of Stock Prices to Earnings and Dividend Announcements," *Journal of Financial Economics,* 13, 223–252.

Pfleiderer, P., 1984, "The Volume of Trade and the Variability of Prices: A Framework for Analysis in Noisy Rational Expectations Equilibria," working paper, Stanford University.

Schwert, G. W., 1989, "Why Does Stock Market Volatility Change over Time?," *Journal of Finance,* 44, 1115–1153.

Shalen, C. T., 1993, "Volume, Volatility and the Dispersion of Beliefs," *Review of Financial Studies,* 6, 405–434.

Shleifer, A., and L. H. Summers, 1990, "The Noise Trader Approach to Finance," *Journal of Economic Perspectives,* 4, 19–33.

Varian, H. R., 1985, "Divergence of Opinion in Complete Markets," *Journal of Finance,* 40, 309–317.

Varian, H. R., 1989, "Differences of Opinion in Financial Markets," in Courtenay C. Stone (ed.), *Financial Risk: Theory, Evidence and Implications, Proceedings of the Eleventh Annual Economic Policy Conference of the Federal Reserve Bank of St. Louis,* Kluwer, Boston, 3–37.

Wang, J., 1992, "A Model of Competitive Stock Trading Volume," working paper, MIT.

Ziebart, D. A., 1990, "The Association Between Consensus of Beliefs and Trading Activity Surrounding Earnings Announcements," *Accounting Review,* 65, 477–488.