The Joint Dynamics and Stability of Stock Prices and Volume

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ABSTRACT

This paper examines the joint process for volume and volatility. Local linear models are used to analyze stability of the joint processes in different regions of volatility-volume space. It is found that this way of looking at the series changes dramatically the results found by using a global linear approach. The estimated model for volume and volatility shows that some small shocks may be temporarily amplified, but they will eventually dissapate.

I. Introduction

An important issue in the analysis of financial markets is the extent to which trade is self-generating. Are markets stable processors of new information, or do they go through periods in which there is some positive feedback in the price and volume processes? Understanding this feature of markets takes us closer to understanding the process through which new information is ascimilated into market prices. Several empirical features suggest that there may be more structure to the price-volume process that needs to be explained. In showing that volatility was larger on days with longer trading periods French and Roll(1986) presented some of the earliest evidence that there may be some self-generation going on in the process of trading. Later papers have shown further structure in the volume and volatility process.[1]

This rich set of empirical features is beginning to show a world which is different from one in which changes in volatility and volume are caused by changes in a single latent variable standing for the speed of information flow. In this far simpler world we would only observe joint persistence, and contemporaneous correlations in the two series. However, this is an important world to keep in mind as a benchmark. If this is the extent of the structure contained in observed finacial series, then attempts at modeling the actual trading process will probably be somewhat futile since they will have very little actual information to constrain the theoretical models.

Several theoretical papers have also begun to attack this problem. In models such as Blume, Easley, and O'Hara(1991), and Romer(1993), trading volume plays an important role in helping traders understand the quality of their information signals. Other papers such Brock and LeBaron(1993) and Wang(1991) also attack the problem of volatility formation with heterogeneously informed agents. Each of these models produces a nontrivial amount of trading volume, and testable implications for the joint stochastic processes.

This paper also draws on some work on nonlinear systems which stresses the importance of local properties over global ones. [2] For deterministic systems one is often interested in the spreading properties of nearby trajectories. These spreading rates are the basis for the sensitive dependence to initial conditions for chaotic systems. However, in systems that are driven by outside signals or outside stochastic shocks the traditional notion of spreading and lyapunov exponents may give very little information about the true nature

[1] Among these are Karpov(1986) and Gallant, Rossi, and Tauchen(1992) who show that price volatility and volume are highly persistent, and contemporaneously correlated. The persistance of volatility itself has generated a large litterature using GARCH/ARCH types of models which is surveyed in Bollerslev et al.(1992) Also, several studies have begun to examine the nonlinear stucture to the dynamics of the price-volume relationships. Hiemstra and Jones(1993) find there are nonlinear relationships between trading volume and volatility that cannot be easily explained through standard models of persistent volatility, and Gallant, Rossi, and Tauchen(1993) display nonlinear impulse reponses for the joint volume-volatility process. Several papers have also analyzed changes in conditional predictability conditioned on trading volume. These include Antoniewicz(1992), Campbell, Grossman, and Wang(1993), Conrad, Hameed, and Niden(1992), LeBaron(1992), and Morse(1980).

[2] For examples see Driesler and Farmer(1989), Nese(1989), and Wolff(1992).

of the system. Input noise may help keep a system moving through locally unstable regions that it might eventually leave forever if the noise were turned off. Such a system has dramatically different properties with and without noise. Other types of systems might involve several basins of attraction where outside noise occasionally kicks the system from one to the other. These problems begin to show the difficulties in characterizing stochastic nonlinear dynamical systems.

There is one common feature linking the earlier mentioned litterature on financial markets and the litterature on dynamical systems. There is a desire to understand the impact of outside shocks to the system. Does the system sucessfully fight new noise coming in, or are there periods when the structure of the system itself amplifies the outside stimulus. Finding, measuring, and quantifying this is not an easy task, but the question is an important one for many areas of economics. The important question is whether fitted nonlinear stochastic systems can be quantified along this dimension in some way. This paper will not answer this question, but some progress will be made to show its importance.

This paper is in 4 sections. Section II gives a brief summary of the data sets used. Section III shows some local spreading estimates, and simulation comparisons. Section 4 runs simulated version of the estimated nonlinear model through impulse response functions and volatility tests. Section 5 will conclude and summarize.

II. Data Summary

The returns series used in this study are the CRSP value weighted index from July 1962 through June 1988. This time period gives 6430 daily data points.[3] From this series a simple volatility series is constructed as the log of the daily return squared,

$$s_t = log(r_t^2).$$

Under the conjecture that returns follow some volatility proccess plus multiplicative noise,

$$r_t = \sigma_t e_t,$$

the log transform will make this an additive noise process. Also, the units will correspond to approximate percentage changes in volatility. This series ignores changes is conditional means, which may be a problem. However, changes in conditional means at this frequency will probably be small relatve to the changes in conditional variance.

[3] The results have been tested to see if they are robust to the removal of the 1987 crash. No major differences were seen.

The volume series is similar to the one used in LeBaron(1992). It is NYSE turnover over the same period as the returns series. Turnover ratios are trending up over time, so this series is then detrended using a 100 day moving average. Finally, the detrended series is log transformed giving,

$$v_t = log(\frac{rv_t}{ma_t}),$$

where rv_t is the turnover at time t, and ma_t is the 100 day moving average of this series. This volume series is an imperfect measure of total trading activity since it aggregates across differently valued shares of traded securities, but it does provide information about the local changes in aggregate trading activity on the New York Stock Exchange.

For both series day of the week effects are estimated and removed from the series using OLS and day of the week dummies. The reason for doing this is to avoid spurious results related to dynamics from the timing of certain macro-economic news items.[4] The residuals of this process will be the series studied for the remainder of the paper. Table 1 gives summary information for the two series. The most striking feature is that both series are correlated over time and this correlation pattern is highly persistent for both volume and volatility. This is consistent with many previous results.[5] There is some amount of negative skewness in the volatility series which is probably due to some near zero return days. This can also be seen in the low minimum value for the volatility series.

Figure 1 displays one other well known feature of these two series. This picture shows some of the cross correlation patterns in the two series. A strong contemporaneous correlation is seen between volume and volatility. Also, the correlation extends to volume one period in the future. Actually, the future correlation is slightly larger than the contemporaneous correlation.

This section replicates many well known facts on volatility and trading volume, showing a large amount of persistence and structure in both series. Far from being random, these series contain a large amount of information for study. The remainder of the paper will look at the dynamics of these series more closely.

III. Stability and Joint Dynamics

Table 2 shows parameter estimates for simple 1 lag VAR for the volatility and volume processes. The results show the strong persistence in both series, and also some amount of dynamic cross correlations. The strongest of these being a negative relation from lagged volume to future volatility.[6]

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[4] These day of the week differences are significant for some days, but their magnitudes are small relative to overall variability in the series. Most of the results in this paper have been replicated on the raw series as well.

[5] See Bollerslev et al.(1992), Gallant, Rossi, and Tauchen(1992), or Karpov(1987).

[6] It should be remembered that this simple system is clearly misspecified given the persistence in both series.

A quick glance at the parameters suggests that the response of this system to new shocks is probably stable. Shocks to the system would be quickly dissapaited. Yao and Tong(1992) suggest one measure t One measure of the overall speed at which new shocks to the system disappear is the largest eigenvalue of $A^T A$. Let λ be the largest eigenvalue of $A^T A$. It follows that the deterministic distance one period in the future will be bounded by,

$$||F(x_t + \delta) - F(x_t)|| \leq |\lambda|||\delta|| + o(||\delta||),$$

where $||.||$ is the Euclidean norm L^2. For this estimated system this number is 0.87. This again shows a strongly stable, and somewhat persistent series, quickly dampening new shocks.

One possibility is that this global linear picture of the system is incorrect. It is possible that the joint dynamics of the two series may depend on the current state of the system. Also, the stability properties of the system may be different in different regions. To explore this, a local linear approximation is used.[7]

To get a general interpretation, and also to facilitate simulations in future section, a very simple form of the local linear fit will be used. The state space (s_{t-1}, v_{t-1}) is split into four subregions using the median value for each series as the splitting level. These four regions will be refered to as high and low volume and volatility respectively. Since the two series are contemporaneously correlated the four regions will not contain the same number of points. There will be more points in the low volume - low volatility region, and in the high volume - high volatility region.

The one lag VAR from table 2 is estimated for each of the four regions in table 3. The results from the first table change dramatically in certain regions. For example, the coefficient of volatility on lagged volume in the low-low region is -1.880, indicating a larger than 1-1 response of volatility on a lagged volume shock. Also, there is some indication of a larger than 1-1 response in the high volatility - low volume region where the coefficient is -1.362. Another interesting pattern that appears is in the high volume region where the lagged volume to volatility channel changes sign, and falls below 1, with parameter estimates of 0.547 and 0.174 in the high and low volume periods respectively. A final pattern that appears is an increased coefficient of volatility on lagged volatility. For the low volatility periods these numbers are quite low, 0.071 and 0.007, for the low and high volume periods respectively. For the low volume period this number is insignificantly different from zero.

The impact of trading volume conditioning information is displayed in table 4 which removes volume from the conditioning information set. This table displays estimated parameters for the same VAR system looking only at low and high volatility periods. The coefficient of volatility on lagged volume for low

[7] This follows procedures used in Brock, Hsieh, and LeBaron(1992), Cleveland(1979), Diebold and Nason(1990), and Farmer and Sidorowich(1987).

volatility is -1.13, which although greater than 1 in absolute value is still much smaller in magnitude from the -1.88, from the previous estimates in the low volume, low volatility period. In general the case for any kind of local instability is much weaker when only volaltility conditioning information is used.

It is not clear whether the presense of some of these apparent local instabilities or is a statistically significant phenomenon. Tables 5 and 6 address this issue by comparing results from the original series with representative simulated series. The local instability is summarized by looking at the local eigenvalue of the estimated VAR matrix, $A^T A$. Table 5 presents estimates for the eigenvalues of the 2X2 VAR matrices and compares these numbers to results from simulations. If the largest real root is greater than one then the local linear system is unstable. Table 5 shows that to sub sections of the series are locally unstable, the low volume section for both low and high volatility. These sections have eigenvalues of 3.8, and 2.3 respectively.

The important question is whether these large numbers are statistically significant. Table 5 presents comparisons of these numbers with simulations. The simulations are based on two different bivariate linear specifications for the joint volatility-volume process. The first tested null model uses the global linear specification estimated from table 2. Residuals from this model are estimated, and the volatlity-volume process is simulated back using the residual pairs scrambled with replacement. Care is taken to draw the pairs simultaneously which maintains contemporaneous dependence in volatility-volume shocks for the simulation. The numbers in parenthesis give the fraction of simulations generating values larger than those from the original series, a simulated p-value. The two largest eigenvalues are unusually large relative to the simulations giving p-values of 0.002, and 0.05.

It is clear from the initial summary table that this simple 1 lag VAR framework will be misspecified. In order to account for this a longer lag model is simulated. Using a Scharz(1978) criterion, a model for volatility is fit with 8 lags on volatility, and 2 on volume, and one for volume is fit with 11 lags on volume and 1 on volatility. A similar simulation procedure is followed where residuals of this model are estimated and pairwise residuals are redrawn with replacement. The dual linear systems are rebuilt into a joint volume and volatility series. The second row under each estimated eigenvalue gives the simulated p-value from this simulation. Again, for the two low volume periods it shows that few of the simulations generated an eigenvalue as large as the original series.

Table 6 repeats these same tests for a finer grid size. The 2X2 grid is replaced by a 3X3 grid. In this case the grid points are determined by the 1/3, and 2/3 quantiles for the two series. This again divides the space into 9 unequal segments. This table repeats the results from the earlier table in demonstrating that some of the periods appear to generate large eigenvalues which can not be replicated by simulating ordinary linear systems. These periods are concentrated in the upper left corner of the figure. If there are any possibilities for instability they are during periods of generally low volume or volatility.

Figure 2 shows estimated eigenvalues over volatility-volume space. These numbers are estimated by moving a band across both volume and volatility space. These numbers are transformed into their fractiles so that they are uniformly distributed across $[0, 1]$. Then a weighted regression is performed where the weighting depends on the distance from the grid point in figure 2. For example, at grid point $(0.5, 0.5)$ points with lagged values close to the median volatility and volume will be weighted heavier than those far from the median. The weighting is done using euclidean distance. This figure shows a smooth and consistent pattern to the instabilities, with the largest values coming from periods of low volume and volatility.

IV. Simulating the Joint Processes

This section presents some simulation results for the joint piecewise linear system estimated in the first section for the volatility-volume process. Each of the four pieces have no potentially unstable paths since in each case volume will clearly be converging to a fixed point.[8] Given that the deterministic system is stable, then the theorems from the appendix by Chan in Tong(1990) will apply and the stochastic system will be geometrically ergodic. Therefore, the simulations that follow will not be influenced by starting conditions.

The first set of simulations seeks to find out how much the nonlinear structure contributes to the overall volatility of the system. This is done by simulating the both the global linear system from table 2, and the 4 part linear system from table 3 using varying levels of stochastic shocks. The shocks used are the residuals estimated from the global linear model. These residuals are scaled by a noise level, α. The standard deviation of both the simulated volatility and volume series are estimated from a simulation of length 6240, and the ratio,

$$\frac{\sigma_\alpha^{NL}}{\sigma_\alpha^{L}}$$

is estimated, where NL corresponds to the nonlinear model, and L to the linear representation.

Figure 3 plots this ratio for varying levels of the noise level, α. For low levels of noise, the nonlinear model boosts the standard deviation by more than a factor of 2 for volume, and almost 50% for volatility. However, as the noise level is increased to 1 the comparison shows the convergence between the linear and nonlinear volatility levels. It is sensible that they should converge eventually, since for large α most of the variability is being driven by the incoming shocks, and model structure eventually should become insignificant. It is interesting that this appears to happen well below $\alpha = 1$, the level of noise from the original series.

[8] Eigenvalues in each simple linear system are less than 1 in absolute value. Each seperate system viewed on its own is stable, but this may not indicate that the entire system is stable. Note also, that the system has four different fixed points corresponding to each of the pieces. The fixed points have not been tested to see which region of the state space they fall in. However, simulations show that the deterministic system may converge to different points depending on its starting values.

Figures 4 and 5 look at impulse response features of the linear versus nonlinear systems. Nonlinear impulse response like objects are used in to analyze the response of the system to various shocks, and what types of local trajectory spreading is present. Two different measures are used. First, the expected distance after a shock is estimated using,

$$E(X^*_{t+j} - X_{t+j}|X_t = x, Y_t = y, X^*_t = x + d_x, Y^*_t = y + d_y).$$

This expectation for the nonlinear system is a nontrivial object. It is estimated using computer simulations. 1000 interations are performed for the system and the perterbed system. For these runs the system and perterbed system are hit with the same shocks at each future time period after time t. This corresponds to exactly the experiment suggested in Nychka et al.(1992) for quantifying chaos in noisey systems where one wants to know if nearby trajectories diverge under the same stochastic shocks. For this measure it is not crucial whether the same shocks are fed into the original and perturbed series. This is due to the linearity of the the distance measure used here. However, for other distance measures it will make a difference.

Figure 1 presents the response of the **volatility** series to shocks equal to $(1, -0.5, 0.5, 1)$ times the volume standard deviation applied to the **volume** series. Also, the impulse respose from the global linear system is presented for comparison. An important feature of nonlinear impulse response functions is that they are dependent on where the system is started. It is clear from the estimated linear systems that it will make a difference if the shock is applied while the system is in the low volume region or the high volume region. Figure 4 starts the system first at the unconditional means for both volume and volatility, and at the lower 25 percentile volume and volatility levels, putting the system well into the low volume, low volatility region.

The responses when the system is started at the mean show large volatility spikes from the volume shocks when the shocks are negative. The level of the reponse is scaled by the magnitude of the volume shock, so for this experiment the volatility can respond by almost a factor of 5 for some levels of volume shocks. The reponse to both volume shocks is much larger than 1. For positive shocks the reponse is very small. The system jumps a small amount, then moves slightly negative at 2 days into the future, and then steadily returns to zero. This dampened response is due to the fact that the shocks pushed the system into the stable region. Also, the linear response can be seen to be quite small relative to the nonlinear responses.

In the bottom panel the system is moved into the low volume-volatility region. In this case the shocks affect the system in a more predictable fashion since they don't move it into another region. The positive shocks cause a fall in volatility, while the negative shocks cause a large boost. The effect of both shocks is much larger than the linear model would predict.

In figure 5 a new measure is used to try to account for the movements of both volatility and volume. The following object is estimated,

$$E(d\left[(X^*_{t+j},Y^*_{t+j}),(X_{t+j},Y_{t+j})\right]|X_t=x,Y_t=y,X^*_t=x+d_x,Y^*_t=y+d_y).$$

where d is the Euclidean distance or L^2. In this case it is important that the same stochastic shocks are fed to the original and displaced series. When the system is simulated from the mean starting location a result similar to last one appears. There is a strong response to the negative shocks, but very small responses to the positive shocks. Again, the nonlinear reponses are much larger than the those from the linear model.

In the bottom panel of figure 5, a common response to all 4 shocks is shown for the linear model. Obviously the symmetry of the distance measure blurs the differering impacts of positive and negative shocks. Once again the shocks are much larger than the those from the linear model.

In each of the previous cases analyzing nonlinear shocks showed several important qualitative features of the estimated nonlinear model. First, the reponse to shocks depends both on the state of the system and on the size and sign of the shock. This sort of feature is not a characteristic of linear impulse response functions. Second, the estimated impact of a volume shock on the system is much larger for the estimated nonlinear model than for the linear system. This suggests that markets have some elements of local instability in which certain features such as volatility bursts may be self-generating. Finally, all the response functions estimated did converge to zero eventually. After 8 or 9 days had past, they were all close to zero. This is a somewhat negative result on the persistence of differences in trajectories for the nonlinear models. Obviously, all the simulations depend critically on the specification fitted here, and they can only be viewed as suggestive about the actual structure contained in these series.

V. Conclusions

This paper has shown that there is evidence for local instabilities in the volatility-volume process. A 10 percent shock to the volume process may be amplified several times in volume over several days in the future. This observed channel is not a feature when linear techniques are used, and simulated linear model cannot reliably replicate this feature. Taking this into account has a clear qualitative impact on some standard diagnostics such as impulse response functions. The nonlinear model used here does appear to contribute to some increased variability in the simulated joint process, but this increase decays as the noise level is increased to a level close to that of the actual data.

These features do not demonstrate that there is deterministic chaos in volume and volatility. They merely show that there is a local instability. Also, it is still not really clear what chaos means in stochastically driven system. Spreads between starting values under the simulations do dissappear eventually showing the power of the stochastic inputs to wash out any differences. By this criteria the system fails the Nychka et al.(1992) criterion. It also should be added that the model used here is best thought of as a demonstration rather than a good model of volume and volatility. This simple framework allows certain charateristics to be clearly demonstrated in the state space. However, this model would most likely fail any test for model specification. It is used to point out a property of the series that is not well captured in a linear framework. Further tests of other specifications, and biases generated by this mispecified model are under way.[9]

Finally, what this says about volume and volatility cannot be ignored. The fitted model says that the two series are persistent and that there is a channel from volume to future volatility. This channel varies dramitically from low to high volume periods. During low periods it is negative, and during high periods it is positive.

These results, although interesting, are still very preliminary. Further tests need to be performed on nonlinear specifications for the joint processes. If they prove to be robust they will assist in the continuing process of trying to understand what drives actual trading and price determination in financial markets.

[9] One important addition will be to improve the volatlity process. Squared returns are a very noisey proxy for actual volatility, and some other representations such as GARCH are being experimented with.

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Table 1
Summary Statistics
Weekly Exchange Rates : Log First Difference

Description	s	v
Sample Size	6240	6240
Mean	0.000	0.000
Std.	2.430	0.236
Skewness	-1.315	0.254
Kurtosis	7.158	4.223
Max	7.738	1.388
Min	-23.130	-1.291
ρ_1	0.120	0.672
ρ_2	0.088	0.532
ρ_3	0.097	0.489
ρ_4	0.116	0.452
ρ_5	0.121	0.419
ρ_6	0.106	0.380
ρ_7	0.112	0.355
ρ_8	0.094	0.348
ρ_9	0.096	0.312
ρ_{10}	0.106	0.302
Bartlett	0.012	0.012

Summary statistics on volatility and volume. Series are from n 1962 through September 1987. Volatility is measured using squared daily returns from the Dow Jones Industrial Index. Volume is daily NYSE trading volume divided by number of shares outstanding (turnover ratio). Volume is detrended using a 100 day moving average. Both series are log transformed, and day of the week effects are removed.

Table 2
Global VAR Parameter Estimates

$$s_t = a_s + b_s s_{t-1} + c_s v_{t-1}$$
$$v_t = a_v + b_v s_{t-1} + c_v v_{t-1}$$

Dependent	a_i	b_i	c_i
s_t	0.000	0.128	-0.651
	(0.030)	(0.012)	(0.128)
v_t	0.000	0.004	0.665
	(0.002)	(0.001)	(0.009)

Global simple VAR estimated over entire time series. s_t is volatility $log(r_t^2)$, and v_t is log trading volume (turnover detrended using 100 day moving average).

Table 3
Local VAR Parameter Estimates

		Low Volume			High Volume		
		a_i	b_i	c_i	a_i	b_i	c_i
Low Volatility	s_t	-0.258	0.071	-1.880	-0.571	0.007	0.547
		(0.104)	(0.026)	(0.391)	(0.119)	(0.032)	(0.493)
	v_t	-0.026	0.002	0.557	0.020	-0.001	0.507
		(0.007)	(0.002)	(0.028)	(0.007)	(0.002)	(0.032)
High Volatility	s_t	-0.538	0.376	-1.362	-0.602	0.389	0.174
		(0.168)	(0.075)	(0.469)	(0.124)	(0.054)	(0.320)
	v_t	-0.038	0.018	0.630	-0.025	0.021	0.674
		(0.012)	(0.006)	(0.035)	(0.010)	(0.004)	(0.026)

VAR's estimated conditionally on lagged volume and volatility information. High and low partitions are set using the sample median (ie. low volume is volume below the median). Numbers in parenthesis are standard errors.

Table 4
Local VAR Parameter Estimates: Volatility Conditioning Only

		a_i	b_i	c_i
Low Volatility	s_t	-0.200	0.043	-1.132
		(0.057)	(0.020)	(0.194)
	v_t	-0.007	0.001	0.623
		(0.003)	(0.001)	(0.013)
High Volatility	s_t	-0.444	0.392	-0.523
		(0.089)	(0.044)	(0.173)
	v_t	-0.029	0.0120	0.685
		(0.007)	(0.003)	(0.013)

Table 5
Local Eigenvalue Estimates

	Low Volume	High Volume
Low Volatility	3.8484	0.5562
VAR(short)	(0.002)	(0.788)
VAR(long)	(0.000)	(0.688)
High Volatility	2.3652	0.5034
VAR(short)	(0.050)	(0.902)
VAR(long)	(0.050)	(0.892)

Eigenvalues are estimated from VAR $2x2$ matrices in each subregion. They are the largest Eigenvalue for $A^T A$ where A is the $2x2$ VAR matrix. Numbers in parenthesis refer to the fraction of simulated runs generating eigenvalues as large as those from the original series. VAR(short) refers to a simulated one lag VAR on volume and volatility. VAR(long) refers to a long lag VAR on volume and volatility where the optimal lag lengths are determined by the Schwarz criteria.

Table 6
Local Eigenvalue Estimates: 3x3 Grid

	Low Volume	Medium Volume	High Volume
Low Volatility	2.9695	9.7600	2.6398
VAR(short)	(0.056)	(0.098)	(0.192)
VAR(long)	(0.054)	(0.074)	(0.120)
Medium Volatility	7.4418	2.4042	0.5095
VAR(short)	(0.002)	(0.456)	(0.854)
VAR(long)	(0.000)	(0.428)	(0.770)
High Volatility	1.1984	0.6310	0.5068
VAR(short)	(0.400)	(0.856)	(0.834)
VAR(long)	(0.366)	(0.862)	(0.864)

Eigenvalues are estimated from VAR $2x2$ matrices in each subregion. They are the largest Eigenvalue for $A^T A$ where A is the $2x2$ VAR matrix. Numbers in parenthesis refer to the fraction of simulated runs generating eigenvalues as large as those from the original series. VAR(short) refers to a simulated one lag VAR on volume and volatility. VAR(long) refers to a long lag VAR on volume and volatility where the optimal lag lengths are determined by the Schwarz criteria.

Table 7
Local Eigenvalue Estimates: Subsamples

	Low Volume	High Volume
	Subsample 1	
Low Volatility	8.9384	0.4993
VAR(short)	(0.000)	(0.990)
High Volatility	1.8846	0.4826
VAR(short)	(0.276)	(0.990)
	Subsample 2	
Low Volatility	2.1824	0.9699
VAR(short)	(0.022)	(0.212)
High Volatility	2.5825	0.6502
VAR(short)	(0.012)	(0.350)

Eigenvalues are estimated from VAR $2x2$ matrices in each subregion. They are the largest Eigenvalue for $A^T A$ where A is the $2x2$ VAR matrix. Numbers in parenthesis refer to the fraction of simulated runs generating eigenvalues as large as those from the original series. VAR(short) refers to a simulated one lag VAR on volume and volatility. VAR(long) refers to a long lag VAR on volume and volatility where the optimal lag lengths are determined by the Schwarz criteria.

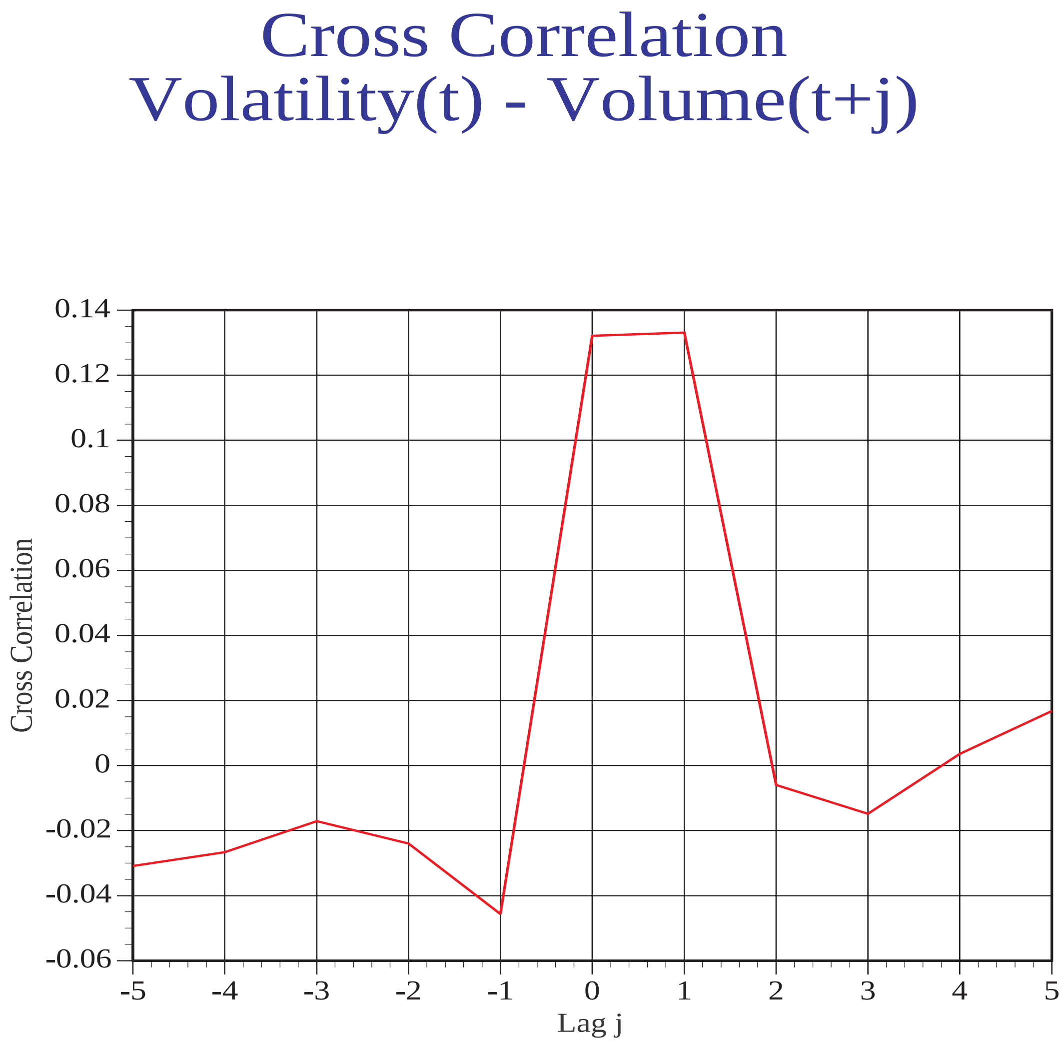

Cross Correlation
Volatility(t) - Volume(t+j)
Cross Correlation
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-5
-4
-3
-2
-1
0
1
2
3
4
5
Lag j

Figure 1

Estimated Eigenvalues

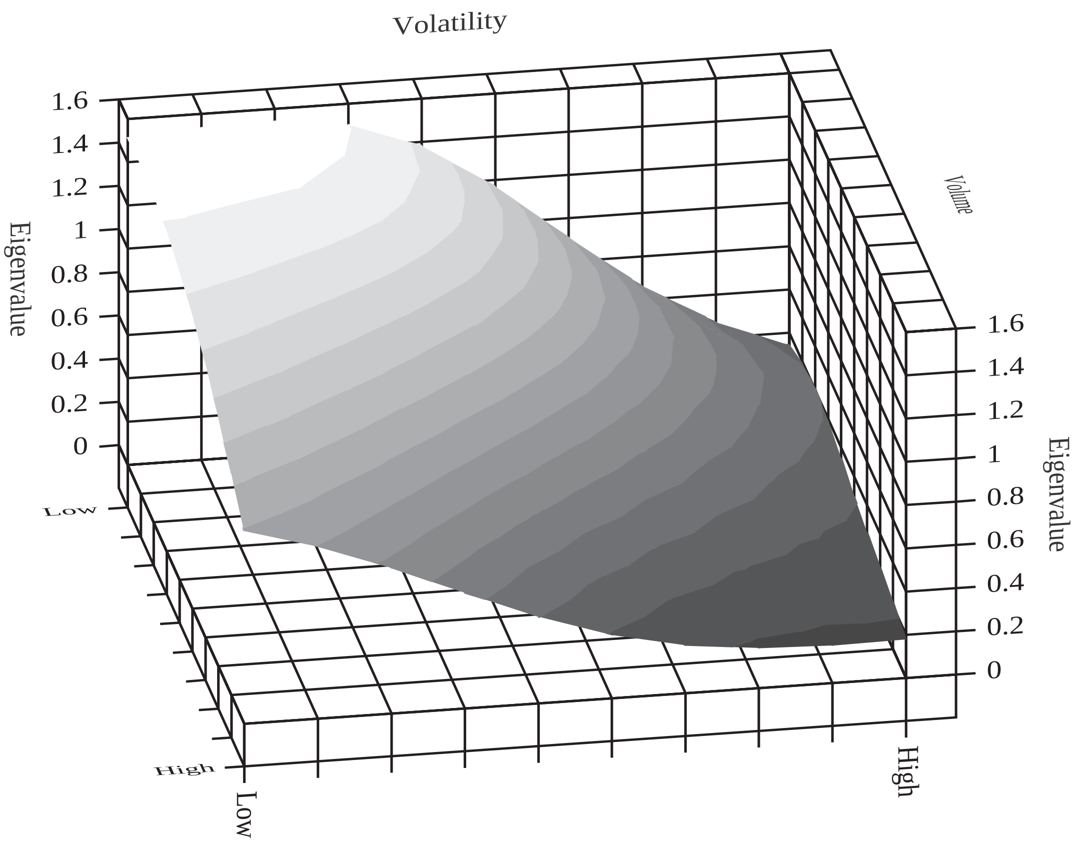
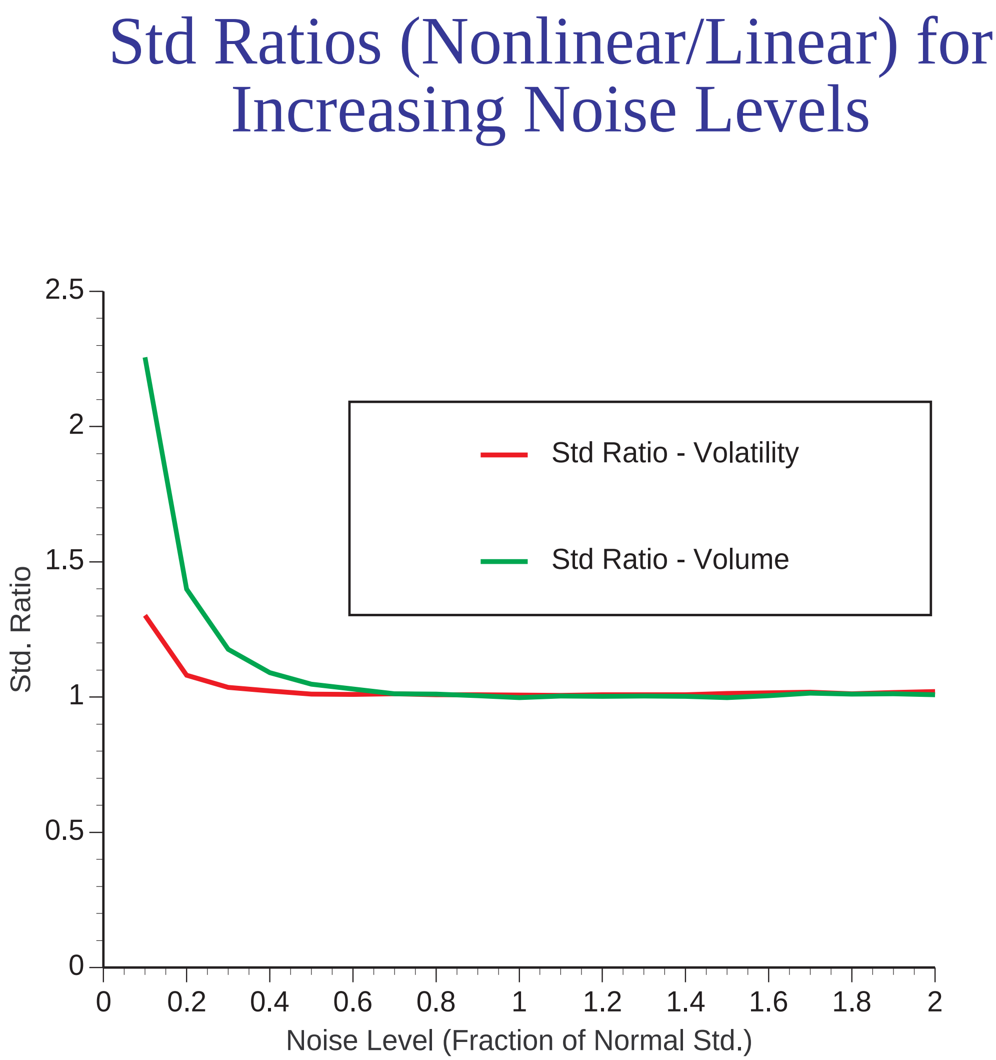


Figure 2



Std Ratios (Nonlinear/Linear) for Increasing Noise Levels
Std Ratio - Volatility
Std Ratio - Volume
Std. Ratio
Noise Level (Fraction of Normal Std.)
0
0.5
1
1.5
2
2.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

Figure 3

Volatility IRF

Response of Volatility to Volume Shocks Starting at
Mean (above) and Unstable Regions(below)

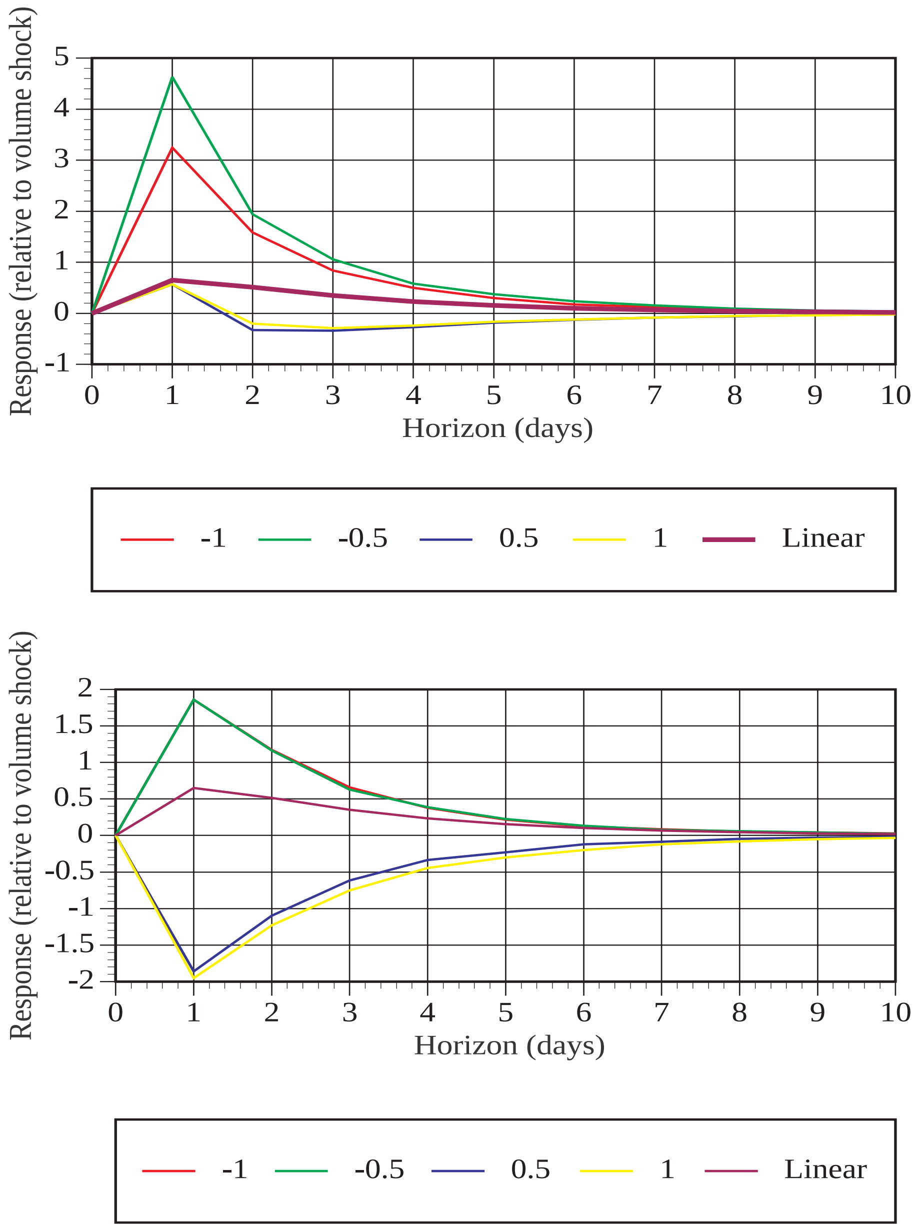


Figure 4

Distance IRF

Vector Euclidean Distance after Volume Shocks starting
at Mean (above) and Unstable Regions(below)

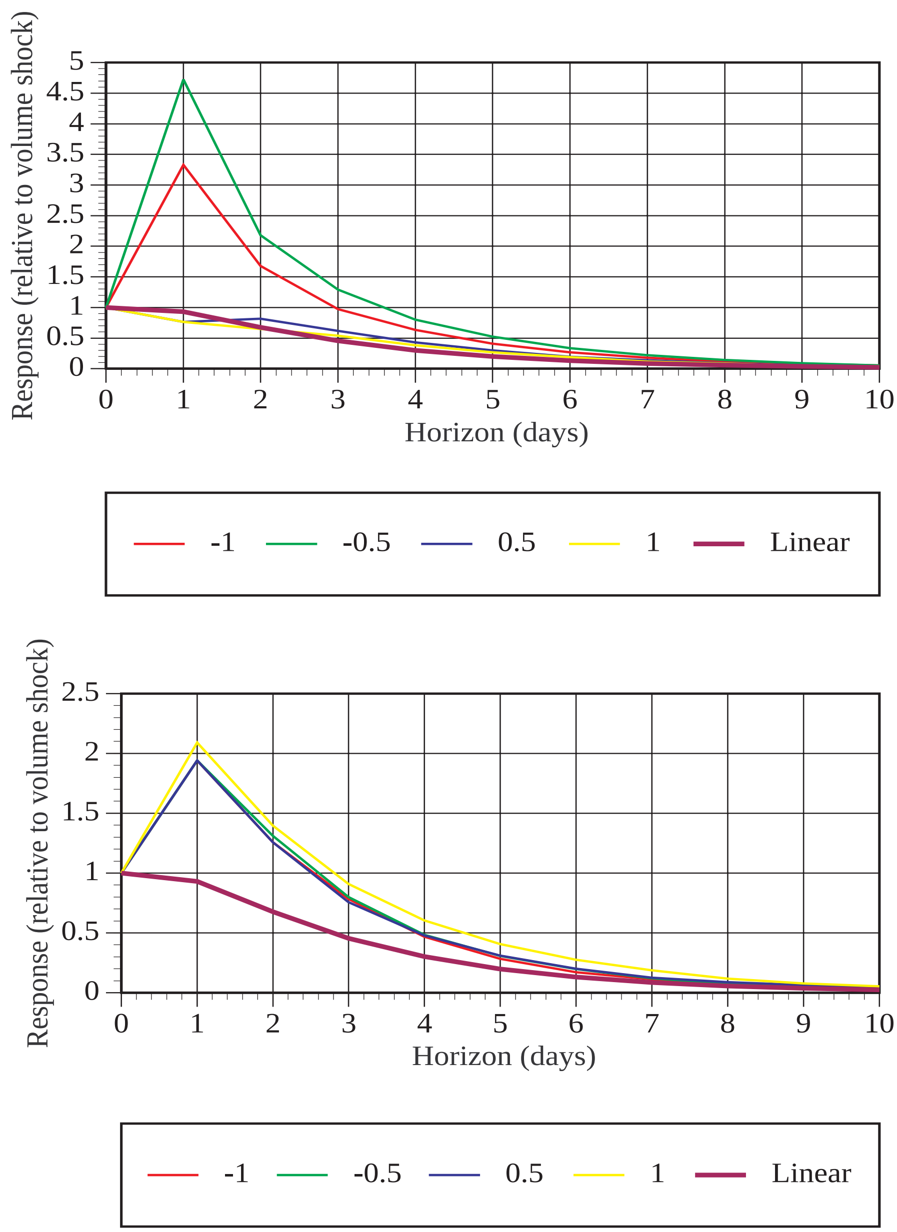


Figure 5